The Process:

Materials: Problem Solving Through Recreational Mathematics by Bonnie Averbach and Orin Chein, Chapter 2 Exercises.

Intended Use: Problems are selected from Chapter 2 of the text. Specific problems will come from the series of question about the Truthtellers and the Liars. These are logic-based problems that will be used to build critical thinking and logical reasoning skills in students. This series is intended for use after the lessons having to do with truth tables and Boolean operations. Through modified gradual release, students will become proficient in reading the problem, identifying required information to solve the problem, and solving the problem.

- For the first 2 (the I do stage), students will be asked to attempt to solve the problem for 5 minutes and then I will walk them through the process.
- For days 3-5, students will be given 5 minutes to think about the problem and then will provide their thoughts as we work through the process.
- For days 6-8, students will be given 3 minutes to think and 5 minutes to collaborate before presenting their solutions.
- On day 9, students will complete the task independently on paper as an assessment of their new skill set (6 minutes).

Intended Outcome: By day 9 (third week), students will be able to apply problem solving skills to the problems and solve the problems on their own.

Gradual Release Process:

1. I do
2. We do
3. You do together
4. You do alone

The Problems

Day 1:

2.1. One summer solstice, while searching for the Southern Shortcut, Silas Seeker, that superman of seafarers, was shipwrecked during a sudden severe squall. Auspiciously, Silas sighted a small sandy isle in the distance and succeeded in swimming ashore. Exhausted, he slipped into a sound sleep. While he slept, Silas dreamt that he was discovered by two trolls, both of whom, fortunately, were able to converse in English. The two were identical in all aspects of their appearance; however, one belonged to a clan whose members always tell the truth, the other to a clan whose members always lie.

“I am Glog and he is Glum,” responded the first. “No, I am Glog and he is Glum,” answered the second.

Just then a third troll appeared. Hoping to shed some light on the matter, Silas pointed to the two and asked, “Which of them can I believe?” “He and I belong to the same clan,” replied the third troll, pointing to the first. “That’s true; they do belong to the same clan,” said the second troll. Assuming that Silas’ dream was accurate, who is telling the truth and what was the name of the island?

Answer: The first was truthful and the island is Hamlock.

Day 2:

2.2. Silas lived on the island for six years (see Exercise 2.1), but during all that time he never learned how to visually tell the difference between members of the two clans. One day Silas met two trolls. The first claimed that they belonged to different clans, whereas the second proclaimed that the first was a liar. To which clan did each of the two belong?

Answer: Troll 1 tells the truth and Troll 2 is a liar.

Day 3:

2.3. During his sojourn on the island, Silas gave names to the two clans (see Exercises 2.1 and 2.2). The truth tellers he called Truthfuls and the liars he called Liars.

One day, Silas met three trolls. He inquired to which clans they belonged. “All of us are Liars,” said the first. “No, only two of us are Liars,” amended the second. “That’s not true either,” corrected the third. “Only one of us is a Liar.” To which clan does each of the three belong?

Answer: Troll 2 tells the truth and Trolls 1 & 3 are liars.

Day 4:

2.4. Shortly after Silas completed building a raft to take him off the island (see Exercises 2.1 to 2.3), five trolls arrived with food and water for his voyage. As usual, Silas inquired about their clans. “Three of us are Truthfuls,” said the first. “That’s right, three of us are Truthfuls,” the second agreed. “No, only two of us are Truthfuls,” said the third. “The first three are all lying,” said the fourth. The answer of the fifth man left no doubt in Silas’ mind as to the clan affiliation of each troll. To which clan did they each belong?
Problem Solving Roadmap—Catherine Tabor

Answer: T1,T2,T3, and T5 are all Liars and T4 is the only Truthful

Day 5:

2.5. One hundred and fifty years after Silas Seeker left the island (see Exercises 2.1 to 2.4), his great-great-granddaughter, Heidi Seeker, found Silas’ diary and decided to revisit the site of her ancestor’s salvation. When she arrived on the island, she was surprised to discover that there were now three clans instead of two. During the century and a half that had elapsed, some intermarriage between the two groups had occurred and the progeny of these mixed marriages comprised a third group of people who always alternated between telling the truth and lying. Thus, when a member of this third group (whom Heidi named Alternators) makes a series of statements, the first statement might be truthful or a lie, but the veracity of each subsequent statement would alternate. In other words, if the first statement were truthful, then the second would be a lie, the third would be true, and so on.

When Heidi first arrived on the island, she encountered a group of three trolls. One was very tall, one was very short, and the third was of medium height. Heidi introduced herself.

“Welcome,” said the shortest of the three. “I am Gladden. He (indicating the tallest of the three) is Lowax. And he (indicating the remaining troll) is Grout.” “He is, as he says, Gladden. But I am Kaut,” corrected the troll of medium height. “And the tall troll is Tildeau.” The tall troll objected: “I am Lowax. But the short one is Waldar. And he (pointing to the troll of medium height) is Gaut.” If at least one of them was a Truthful, what were the names of the three trolls?

Answer: Troll 3 (Lowax) is a Truthful, Troll 1 (Walder) is an Alternator (lying first), and Troll 3 (Gaut) is a Liar.

Day 6:

2.6. Later that day, Heidi (see Exercise 2.5) met another group of three trolls—one from each clan. Heidi asked to which clans they belonged. The spokestroll of the group replied: “I am a Liar; the lady is a Truthful; the young gentletroll is a mixed breed.” To which clans did each belong?

Answer: The spokestroll is an Alternator, the lady is a Truthful, and the young troll is a Liar.

Day 7:

2.7. After Heidi (see Exercise 2.5) had been on the island for a few days, she realized that each individual’s name was woven into his or her clothing and easy to see. When Heidi was introduced to Winken, Blinken, and Finken, she asked her usual question. The replies she received were: Winken: I am an Alternator. Blinken: Winken is a Liar. Finken: Winken is a Truthful. If no two of the three belonged to the same clan, to which did each belong?
Answer: Winken is a Liar; Blinken is a Truthful; and Finken is an Alternator who is lying.

Day 8:

2.8. When Heidi (see Exercise 2.5) asked her usual question to Hocus, Pocus, Crocus, and Cloy, only Hocus and Pocus replied. “I am a Truthful; Pocus is a Liar; Crocus is a Liar; Cloy is a Truthful,” was Hocus’ response. “No, Hocus is a Liar; I am a Truthful; Crocus is an Alternator; Cloy is a Truthful,” corrected Pocus. To which clan did Hocus, Pocus, and Crocus belong?

Answer: Hocus is a Liar, Pocus is an Alternator, and Crocus is a Truthful.

Day 9:

2.9. Each year, the leaders of the three clans (see Exercise 2.5) participate in an athletic competition to determine which of the three should govern the island for the new year. After this year’s competition, the following statements were made by the competitors:

Ange: I came in second; Benge came in last.
Benge: Ange came in last; Conge won.
Conge: I came in last; Ange won.

What was the order of finish in the competition?

Answer: Conge, Ange, Benge