Meaning Coding and Linguistic Analysis of Teacher and Student Topic-Specific Knowledge of Lower Secondary School Mathematics

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Purpose of Study

- Research on teacher knowledge initiated by work of Shulman (1986) has focused on teacher knowledge as a major predictor of student learning and achievement.
- Since then the field benefited from numerous studies (Hill, Shilling, & Ball, 2004; Hill, Ball, & Schilling, 2008; Baumert et al., 2010) that substantially advanced the conceptualization of teacher knowledge and its different categories.

Purpose of Study (cont.)

- Some scholars (e.g., Chapman, 2013; Izsak, Jacobson, & de Araujo, 2012) examined different facets of teacher knowledge without explicitly emphasizing its connection to student learning.
- Other scholars stressed the importance of the kind of knowledge a teacher possesses because it impacts his/her teaching (Steinberg, Haymore, and Marks, 1985). Another line of research (e.g., Hill, Rowan, & Ball, 2005; Baumert et al, 2010; Author, 2011) specifically targets the effects of different types of teachers' knowledge on student achievement.
- There is a need in the field for extending the latter line of research to the level of topic-specific connections between teacher and student knowledge.

Research Questions

Considering the importance of topic-specific knowledge, this study was guided by the following *research questions*:

- to what extent are teachers' topic-specific content knowledge related to student performance?
- how are teachers' mastery of cognitive types of content knowledge associated with the students' topic-specific knowledge?

Conceptual Frame

- There are varieties of frameworks to study cognitive types of teacher knowledge in the literature proposed by different teams of scholars.
- The TEDS-M (IEA, 2012) study of pre-service teachers used the framework consisting of three types of knowledge: 1) knowing, 2) applying, and 3) reasoning.
- The DTAMS instrument designed to measure teacher knowledge (Saderholm, Ronau, & Brown, 2010) along with memorization, conceptual understanding, and higher-order thinking, included pedagogical content knowledge as a separate type.

Topic-specific Content Knowledge: Division of Fractions

- Division of fractions is one of the topics in lower secondary school mathematics curriculum for grade 6th in Russia where the study was conducted (Ministry of Education and Science of Russian Federation, 2004).
- Scholars (Ball, 1990; Ma, 1999) found that teachers have limited topic-specific content knowledge and they lack a conceptual understanding of the topic.
- One of the main reasons is that the topic of the division of fractions is traditionally taught by using the “flip and multiply” or “cross-multiply” procedure (e.g., invert-and-multiply algorithm) without helping learners to understand why it works (Siebert, 2002).

Division of Fractions (cont.)

- Initially, Fischbein et al. (1985), and Simon (1993) have identified two main models for the division of fraction: quotitive (measurement) and partitive (part-to-whole).
- Greer (1992) within the partitive model proposed “rectangular area” model, which later Ma (1999) included in a separate category - “product and factors”.
- Thus, according to Ma (1999, p. 72), there are three main models to represent the division of fractions: measurement, partitive, and product and factors.

Research Design

Teacher Content Knowledge Survey - TCKS (N=97)

Criteria for the Purposive Teacher Sampling:

- teachers represent upper and lower quartiles of the total scores on the TCKS
- have similar teaching experience
- have similar teaching assignments
- teach at similar school settings

Teacher Interview

Teacher - 1 (Irina)

Teacher - 2 (Marina)

Student group -1 (n=29)

Student group -2 (n=26)

Student Problem Solving

Participants

- The selected study sample consisted of two teachers - Irina and Marina (anonymous) - who met the requirements of the purposive sampling.
- Irina and Marina's total scores on TCKS with corresponding quartiles' ranges are presented in table below.

Teacher	Score	Quartile	Range
Marina	25	Q4	24-27
Irina	17	Q1	13-18

Interview and Problem Solving Questions

Question Types	Questions	Teachers	Students
Questions on pedagogical content knowledge	1) When you teach fraction division, what are important procedures and concepts your students should learn? 2) What is the meaning(s) of the division of fractions?	+ +	
Questions on cognitive types of content knowledge	3) What is the fraction division rule? 4) Divide two given fractions and ½ : 1 ¾ . 5) Construct a word problem for the fraction division from the previous question. 6) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (a, b, c, and d are positive integers) ever true?	+ + + +	 + + +

We purposefully used similar questions for teachers and students in order to trace linguistic, procedural, and conceptual traits in their reasoning as well as to analyze non-parametric quantitative associations between teacher and student topic-specific knowledge.

Data Analysis

- In order to respond to the research questions, we conducted meaning coding and linguistic analysis (Kvale & Brinkmann, 2009) of teacher narratives and student responses as a primary method of analysis.
- **The linguistic analysis** technique was employed to specifically check teacher and student use of mathematical terminology (e.g., questions 1-4).
- The data-driven **meaning coding** technique was used to analyze teachers' and students' responses on questions tapping into their understanding of meanings of the division of fractions (e.g., question 5) as well as their justification for solving the non-routine problem (e.g., question 6).

Data Analysis (cont.)

- Considering ranked/ categorical nature of the quantitative data collected in the study, we employed a non-parametric technique.
- To compare two or more samples/ groups on a response variable that is categorical in nature, it is recommended to use the independent-samples Chi-square test (Huck, 2004, p. 463).
- This test helps to detect group differences using frequency data. More specifically, we employed Chi-square statistic to look at two independent groups of students of participating teachers (Irina and Marina's groups) compared on responses to questions on the division of fractions.

Results - Teachers

Teachers' performance on the TCKS by cognitive types of knowledge is presented in table below

Teachers' name	Knowledge of facts and procedures (%)	Knowledge of concepts and connections (%)	Knowledge of models and generalizations (%)
Irina	80	46	30
Marina	90	69	70

Results - Teachers (cont.)

The question 1: when you teach fraction division, what are important procedures and concepts your students should learn?

IRINA	MARINA
Before introducing the fraction division, I would like my students to recall the topic on factoring a polynomial, recall the rule of fraction multiplication, and recall reciprocals. After the lesson on fraction division, I expect my students to know fraction multiplication and division rules, acquire skills to use these rules in standard situations, as well as apply factorization of polynomials.	When I teach fraction division, first of all, I expect students to learn fraction division rule as it applies to the case of common fractions. Then, I expect them to know how to apply the rule to mixed fractions. Further, students need to understand how to use the fraction division in routine and non-routine problem-solving situations. Pedagogy wise, I always support students' motivation through engaging students in small group work and classroom discussion.

Results - Teachers (cont.)

The question 2: what is the meaning(s) of the division of fractions?

IRINA	MARINA
Well… there are two main problems in school arithmetic: finding a part of a whole and finding a whole knowing its' part. Said that the meaning of fraction division is finding a whole knowing its' part.	From my perspective… There are at least two meanings for the division of fractions. The first meaning is based on the interpretation of division as operation opposite to multiplication. In other words… to divide a fraction A by a fraction B means to find a fraction C such as A=B x C. For example, 5/6 divided by 1/6 means that there is a fraction C such as 5/6 = 1/6 x C. Or 5/6 ÷ 1/6 =5. On the other hand, the division is a kind of "sorting". For instance,1/2 = 1/4 + 1/4 = 1/4 x 2 meaning that 1/4 goes 2 times into 1/2 whereas 1/2 goes 1/2 times into 1/4. Hope it makes sense… [smiles]

Results - Teachers (cont.)

The question 3 was the following: what is the fraction division rule?

IRINA	MARINA
The rule of fraction division is reduced to the rule of fraction multiplication. Therefore, you need to multiply the first fraction by the reciprocal of the second one. INT: What do you mean by reduced to fraction multiplication? As students say, cross multiply fractions.	In order to divide fractions, you need to multiply dividend by the reciprocal of the divisor. For example, $\frac{15}{4} \div \frac{3}{10} = \frac{15}{4} \times \frac{10}{3} = \frac{25}{2}$ (writes on a scratch paper). Generally speaking, fraction division "boils down" to multiplication.

Results – Teachers (cont.)

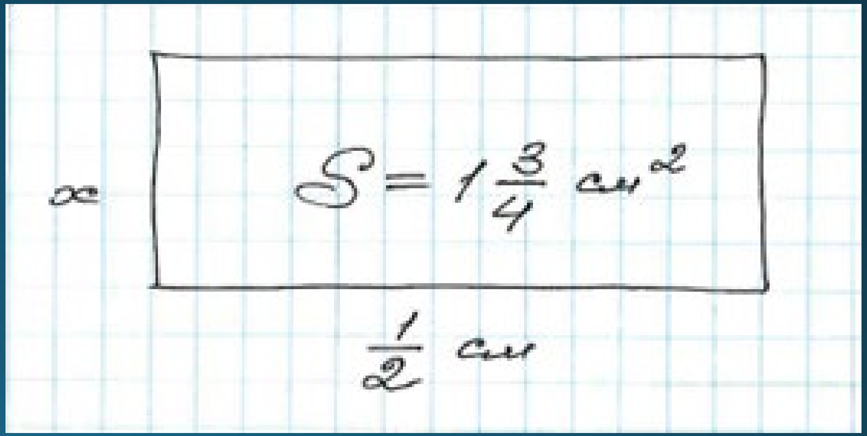
The question 4 focused further on the procedural knowledge: divide two given fractions.

	IRINA	**MARINA**
1 2 3 4 5 6 7	(writes on a scratch paper without any comments) $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3.5$	First, we convert given mixed fraction 1 3/4 to common one 7/4. Notice, here the numerator is larger than the denominator. Then, we replace division by multiplication reversing the divisor. Hence, $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2}$ (writes on a scratch paper).

Results - Teachers (cont.)

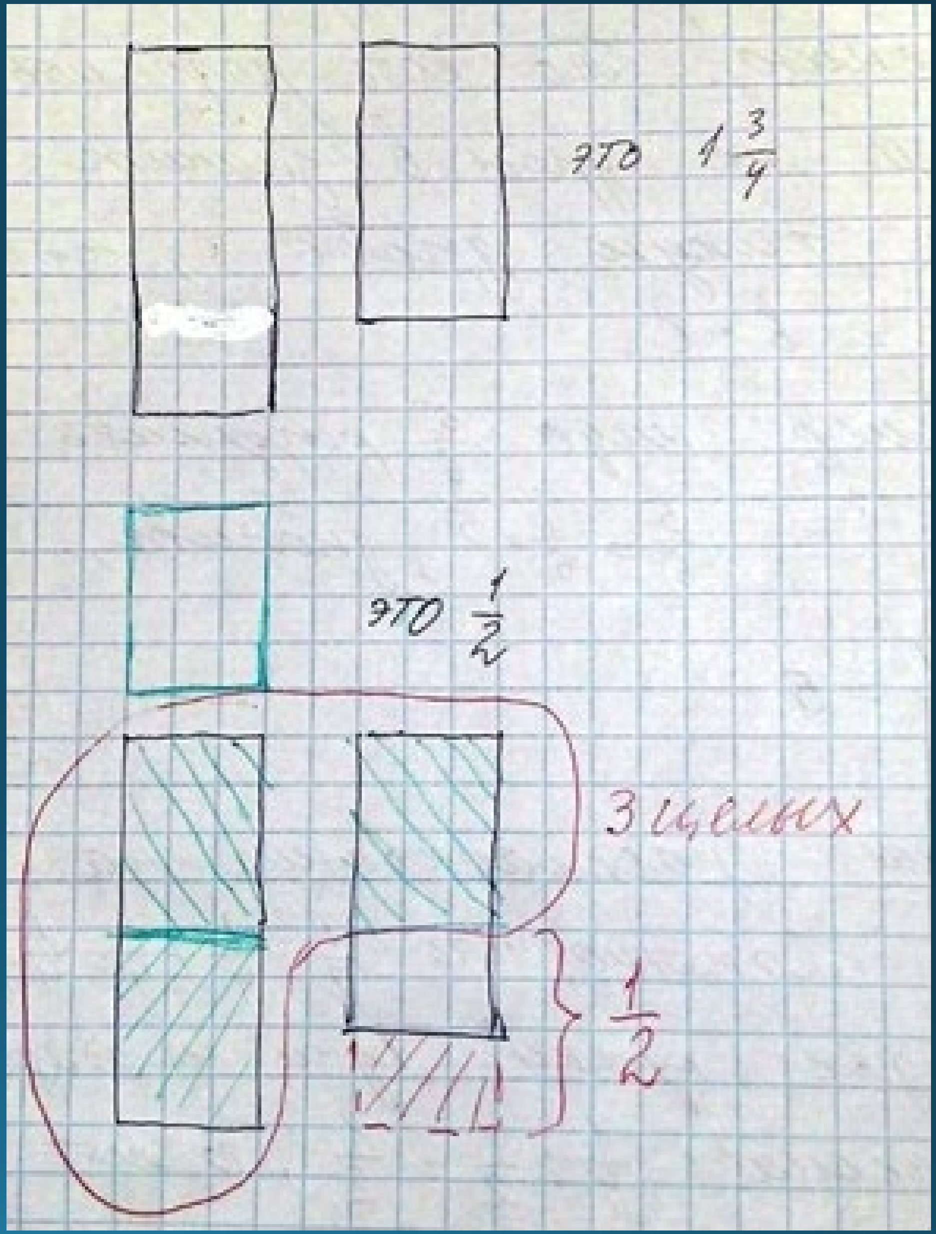
The question 5 asked participants to construct a word problem for the fraction division from the previous question.

IRINA:	May I write down the problem on the paper?
INT:	Yes, of course.
IRINA (writes on a scratch paper):	Area of a rectangle is equal to 1 3/4 cm^2, its length is equal to 1/2 cm. Find the width of the rectangle.



Results - Teachers (cont.)

MARINA:	Here is my word problem: an automated machine packs butter in ½ kg bricks. How many bricks can one make out of 1 ¾ kg of butter? May I draw a picture!?
INT:	Sure.
MARINA:	(draws a picture on a scratch paper)



Results - Teachers (cont.)

The question 6 was aimed at teacher generalized knowledge: is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$

(a, b, c, and d are positive integers) ever true?

#	IRINA	MARINA
1 2 3 4 5 6	The given statement is not correct. In order to divide fractions, you need to multiply the first one by a reciprocal of the second one.	I like this question. It makes me think. INT: Good. Alright, notice that in order to solve this problem *ac/bd* should be equal to *ad/bc*. Right? INT: So... Therefore, *c/d=d/c*. This is possible only if *c=d*.

Results – Students

- Almost all of students in both groups correctly solved the given fraction division problem: 28 out of 29 students - in Irina's group, and 25 out of 26 students - in Marina's group.
- Frequency of terms used by students while solving the fraction division problem is presented in the table below

Terms used by students	Irina's Group (n=29)	Marina's Group (n=26)	χ^2
Flip	19	15	0.356
Cross-multiply	14	10	0.537
Reciprocal	1	5	3.513
Dividend	0	4	4.811*
Divisor	2	8	5.252*
First fraction	10	1	8.042**
Second fraction	11	3	5.033*

Results – Students (cont.)

- Table of frequencies of meanings used by students in constructing word problems along with the chi-square values is presented in table below

Meaning of fraction division used by students	Irina's Group (n=29)	Marina's Group (n=26)	χ^2
Measurement interpretation	3	8	3.574
Finding a whole given a part	1	3	1.331
Missing factor problem	4	3	0.063
Replacing division scenario by multiplication	4	1	1.641

Results – Students (cont.)

Frequencies of student responses along with the chi-square values are depicted by table below.

Student responses	Irina's Group (n=29)	Marina's Group (n=26)	χ^2
Never true	25	6	22.214**
Impossible	3	1	0.859
True if a=b=c=d=1	0	3	3.539
True if a=b and c=d	1	2	0.479
True if c=d	0	8	10.442**
No solution	0	4	4.811*

Results – Students (cont.)

- The number of correct students' responses on questions 1-3 along with the chi-square statistic comparing student performance between groups on each question is presented in table below.
- Overall, the difference between two groups on all three questions was statistically significant (χ^2=*7.644, p*<*.05*)

	Irina's Group (n=29)	Marina's Group (n=26)	χ^2
Question 1	28	25	.410
Question 2	8	14	3.939*
Question 3	1	10	8.430**

Discussion: Teacher and Student Procedural Knowledge

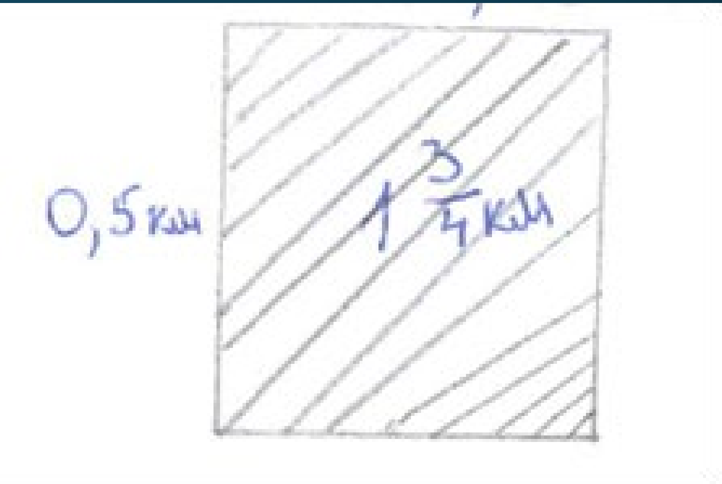
- Even though Irina and Marina had similar mean scores on items measuring teacher knowledge of facts and procedures, the use of mathematical vocabulary and terminology was more accurate in the case of Marina.
- She used mathematically accurate terms “dividend” and “divisor” in answering the question 2, whereas Irina used less accurate terms “first fraction” and “second fraction” which was later reflected in students’ responses: there was a statistically significant difference in the use of terms “dividend” vs. “first fraction” and “divisor” vs. “second fraction” between the groups.
- More specifically, students in Marina’s group preferred to use mathematically accurate terms “dividend” and “divisor” compare to terms “first fraction” and “second fraction” most frequently used by students in Irina’s group.
- This finding could serve as an evidence for the following claim - “The accurate use of the vocabulary is an effective measure of conceptual understanding” (Murray 2004, p. 5).

Discussion: Teacher and Student Conceptual Knowledge

- Based on teacher responses to questions 2 (meaning of fraction division) and 5 (make up a word problem including teachers' drawings) both Irina and Marina demonstrated an understanding of different meanings of division of fractions such as measurement, partitioning, missing factor problem (Flores, 2002; Ma, 1999) that are conceptually equivalent.
- However, we observed an insightful difference in teachers' representation of the selected meanings. While Marina's representation accurately depicted the measurement meaning of division of fractions, Irina's representation was leaning toward procedural approach: her mechanical application of the procedure led to an inaccurate representation of the missing factor meaning of the division of fractions

Discussion: Teacher and Student Conceptual Knowledge (cont.)

- While analyzing one of Irina's student work we found a similar misconception (see figure).
- The student came up with the following word problem using the same missing factor interpretation of the division of fractions: "Area of a land is equal to 1 ¾ km and one of its side – ½ km. Find the width."
- Closely inspecting her word problem and drawing, one could observe two procedural inaccuracies. First, the student mistakenly depicted the area unit as *km* instead of *km²*. Second, she drew a square instead of a rectangle the missing side (3 ½ km) of which should be longer than the known one (½ km).



Discussion: Teacher and Student Generalized Knowledge

- We observed a distinct difference in the approaches used by teachers toward the non-routine question: while Marina recognized and accepted the challenge, Irina ignored and avoided it.
- Our observation also showed Irina's insecurity with questions probing her conceptual understanding of the topic on the division of fractions.
- This observation echoes with the difference between Irina's and Marina's mean scores on the TCKS measuring their conceptual and generalized knowledge.
- Most significantly the difference in teacher knowledge was associated with their students' performance in solving question 3: there was a statistically significant difference between groups at the level of *p-value* $<.01$ in selecting "*never true*" (in favor of Irina's group) and "*true if c=d*" (in favor of Marina's group) responses.

Discussion: Teacher and Student Generalized Knowledge (cont.)

- Another insightful observation on the statistically significant difference between groups in number of blank responses (no solution provided) to question 3: four students in Marina's group decided do not offer any solution (perhaps, due to a doubt) instead of proving any solution (most likely, the incorrect one) compare to none of those cases in Irina's group: all students in Irina's group except one confidently provided incorrect solutions.
- Synthesizing the discussion, we could report that qualitative data collected and analyzed from teacher interviews and students problem solving supported quantitative findings obtained from the TCKS instrument to suggest the following: what a teacher knows is associated with student knowledge in the topic-specific context.

Contribution to Field

- Main results of this study revealed the fact that cognitive types of teacher's content knowledge are connected to the students' topic-specific knowledge reflected in their problem-solving performance as well as the diversity of meanings expressed and language used by students while responding to questions on the division of fractions.
- Thus, findings of this study contribute to the body of research claiming that teacher content knowledge is critical for student learning (Baumert et al., 2010; Hill, Shilling, & Ball, 2004) with a narrow focus on a topic-specific knowledge.

Implications

- Stotsky (2015) claims that "teachers cannot teach what they do not know" (p. 11).
- Therefore, main findings of this study could be used to encourage teachers to attend professional development courses and enhance their content-specific preparation on different cognitive types: procedures and facts, concepts and connections, generalizations and models.
- Moreover, results of this study could be used in designing teacher preparation programs to improve teachers' knowledge about specific topics in order to assist them in establishing effective classroom practices that aim at improving students' topic-specific knowledge.

Future research

- This study focused on the topic of the division of fractions.
- Further research is needed to identify topic-specific connections between teacher and student knowledge on other domains of lower secondary mathematics curriculum.
- Another unresolved question is concerned with the association between teachers' pedagogical content knowledge and student performance in the topic-specific context.
- This research was correlational by its design using a methodology that compared teacher and student responses to a set of identical questions. Potential future research could focus on the cause-and-effect relationship between teacher and student topic-specific knowledge using a similar methodology in the context of the intervention study.

Limitations

- Main limitations are concerned with the teacher sample size and the multiple-choice format of the teacher content knowledge survey.
- One of the complexities in assessing teacher knowledge (Schoenfeld, 2007) is related to limitations of the multiple-choice format in test construction and assessment of teacher knowledge (p. 201).
- Responding to this limitation, we designed the qualitative phase to provide a closer examination of teacher knowledge and understanding and its connection to student knowledge.
- We are also cognizant that classroom observations could be another source of data in this study. However, to explicitly address the research questions we purposefully focused the study on the link “teacher knowledge-student knowledge” in the topic-specific context.
- Based on these limitations, we are sensitive enough to not overgeneralize the study findings.

Conclusion

- The main results of this study open an opportunity to discuss the importance of different cognitive types of the topic-specific teacher knowledge and its potential impact on student knowledge and learning.
- Another promising finding of the study – qualitative analysis of data obtained through teacher interviews and student problem solving suggests that student knowledge may relate to their teachers' knowledge with regard to meanings expressed and language used.
- Congruently, a teacher with a topic-specific content knowledge that is connected, conceptual, and generalized could provide a broader spectrum of topic-specific learning opportunities for his/her students.

Process of publication

- The paper is published in the Journal of Mathematical Behavior (SJR Q1, impact factor=0.97, h-index=34)
- The manuscript was submitted on May 5, 2016
- First response from the editors came back on September 21, 2016
- The manuscript went through two rounds of revisions (3 reviewers, 39 (!) comments)
- The revised manuscript (first round) was submitted back to editors on March 20, 2017
- The second round of revisions were submitted on June 24, 2017
- The manuscript was accepted on June 28, 2017
- The manuscript was published online on July 11, 2017 – on Mourat's birthday ☺
- The manuscript was published in the journal issue on September 2017
- The total time from first submission to the publication – 16 months (!)

Major references

- Ball, D. L. (1990). The mathematical understanding that preservice teachers bring to teacher education. *Elementary School Journal, 90*, 449–466.
- Baumert, J., et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47(1),* 133-180.
- Flores, A. (2002). Profound understanding of division of fractions. In *Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook.* B. Litwiller & G. Bright (Eds.). Reston, VA: National Council of Teachers of Mathematics, 237–246.
- Hill, H., Schilling, S., & Ball, D. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105, 11–30.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal 42*(2), 371–406.
- Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education. 39(4)*, 372-400.
- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Erlbaum.
- Tatto, M. & Senk, S. (2011). The mathematics education of future primary and secondary teachers: Methods and findings from the Teacher Education and Development Study in Mathematics (TEDS-M), *Journal of Teacher Education*, 62 (2), 121-137
- Tchoshanov, M. A. (2011). Relationship between teacher content knowledge, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics, 76(2),* 141-164.

Division of Fractions (cont.)

- In order to develop students' knowledge and comprehension of fraction division teachers themselves need to understand underlying meanings of the algorithms and procedures (Ball, 1990) to make their mathematical knowledge connected and conceptual (Ma, 1999).
- To be connected topic-specific teacher knowledge should address different *cognitive types*: knowledge of facts and procedures, knowledge of concepts and connections, and knowledge of models and generalizations (Tchoshanov, 2011).
- Memorization and application of basic mathematical facts, rules, and algorithms to solve routine problems are required for knowledge of facts and procedures. For example, if a teacher is able to apply fraction division rule to divide two given fractions, then one could say that he/she has procedural knowledge of fraction division.

Division of Fractions (cont.)

- Conceptual understanding requires quantity and quality of connections between mathematical procedures and ideas.
- For instance, if the teacher, who was able to divide two given fractions, can make up a story for the given fraction division problem or can illustrate the division using diagram/ manipulatives, then one could say that he/she possesses a conceptual understanding of the meaning of fraction division.
- Next type of knowledge focuses on generalization. It includes conjecturing, generalizing, proving theorems, etc.
- For example, the teacher could be asked to extend his/her knowledge to the next cognitive type – generalization - and justify whether the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ *(a, b, c, and d* are positive integers) is ever true.

Research Design (cont.)

- Two teachers were selected for the study.
- Teachers completed the Teacher Content Knowledge Survey (TCKS) before teaching a topic on the division of fractions. The TCKS consisted of 33 items measuring teachers' knowledge of facts and procedures (10 items), knowledge of concepts and connections (13 items), and knowledge of models and generalizations (10 items).
- Teachers were also interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge.
- After teaching the topic on fraction division, students of the participating teachers were tested using similar items measuring students' knowledge of procedures, concepts, and generalizations.
- The cross-case examination was performed using meaning coding and linguistic analysis techniques (Kvale & Brinkmann, 2009) to report connections between teacher and student knowledge of fraction division.

Conceptual Frame (cont.)

The cognitive classification used in this study was built on the existing research and consist of:

- knowledge of facts and procedures (e.g., procedural knowledge),
- knowledge of concept and connections (e.g., conceptual knowledge), and
- knowledge of models and generalizations (e.g., generalized knowledge).

Research Design

- The interpretive case study was selected to analyze the topic-specific connections between teacher and student knowledge of lower secondary mathematics.
- According to Merriam (1998) case studies could be classified by overall intent of the study as descriptive, interpretive, and evaluative (pp. 38-39).
- While a descriptive case study presents “a detailed account of the phenomenon under study” (Merriam, 1998, p. 38) and an evaluative case study focuses on “description, explanation, and judgement” (p. 39), the selected interpretive case study provides rich and thick description of data “with the intent of analyzing, interpreting, or theorizing about the phenomenon” (p. 38).

Participants

The study participants were selected using non-probability purposive sampling technique based on the following set of criteria:

- selected teachers should represent upper and lower quartiles of the total scores on the TCKS;
- selected teachers should have similar teaching experience;
- selected teachers should have similar teaching assignments;
- selected teachers should teach at similar school settings.

Participants (cont.)

- Both selected subjects are experienced lower secondary mathematics teachers and both of them are females of the same ethnic origin. Irina has 33 years of teaching experience and Marina - 21 years of teaching experience.
- Addressing the observable difference (33 and 21) in teaching experience between two participants we could refer to existing research that reports the following – “experience did not show a significant association with CK, nor with PCK” (Olfos, Goldrine, Estrella, 2014, p. 927).
- Participants have similar teaching assignments - 5-8 grade mathematics with content addressing the following main objectives: Arithmetic, Algebra and Functions, Probability and Statistics, and Geometry.
- They both teach at urban public schools with similar student population concerning its’ ethnic distribution and SES level.

Participants (cont.)

- The cluster sample of N=55 6th-grade students of participating teachers (29 students in Irina's group and 26 students – in Marina's group) was used for collecting student-level data after they studied a topic on the division of fractions.
- Both teachers were teaching mathematics to the participating cohorts of students for the second consecutive year starting in 5th-grade. Therefore, one may say that they established a certain teaching and learning "history" with these students.
- The topic was a part of the chapter on operations with rational numbers placed in the 6th-grade mathematics curriculum at the beginning of the fall quarter (Ministry of Education and Science of Russian Federation, 2000 and 2004).

Data Sources

- The study used the following data sources:
  - structured teacher interviews on the topic of division of fractions;
  - student data on solving three tasks related to the topic of the division of fractions.
- Teachers were interviewed using two sets of questions related to the topic of fraction division:
  - the subset of questions 1)-2) tapping into teachers' pedagogical content knowledge and aiming at teachers' understanding of learning objectives for the topic of fraction division, and
  - the subset of questions 3)-6) focusing on teachers' possession of cognitive types of content knowledge

Data Sources (cont.)

- Students' written work on solving three tasks related to similar questions on the division of fractions (questions 4, 5, and 6 from table above) was collected and analyzed to examine connections to teacher knowledge.
- We purposefully used similar questions for teachers and students in order to trace linguistic, procedural, and conceptual traits in their reasoning as well as to analyze non-parametric quantitative associations between teacher and student topic-specific knowledge.

Results - Students

- A distinctive feature of the methodology we used in this study, as it was mentioned by one of our reviewers, was that fact that we asked teachers and students to solve and explain their solutions to the same problems.
- Said that, we presented groups of 6th-grade students of participating teachers (Irina's group had n=29 students and Marina's group n=26) with a subset of questions corresponding to different cognitive types of content knowledge identical to those presented to teachers: (1) divide two given fractions 1 ¾ : ½; (2) construct a word problem for fraction division from the previous question; and (3) is the given statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ (*a, b, c, and d* are positive integers) ever true?

Results – Students (cont.)

- The student question 2 was identical to the teacher question 5: construct a word problem for the fraction division from the previous question.
- Less than third of the students (8 out of 29) in Irina's group were able to construct correct story problem for the given fraction division.
- In contrast, more than 50% of students in Marina's group (14 out of 26) came up with correct word problems to the given fraction division.
- Students used different meanings of fraction division to construct their word problems. Analysis of student work revealed that the most frequent meaning used by students in both groups were "measurement", "missing factor problem", and "finding a whole given a part" interpretations of fraction division (Flores, 2002; Ma, 1999).

Results - Students (cont.)

- An example of students' response using “measurement” interpretation – “Mom bought 1 ¾ kg of sugar to make cakes. How many cakes she could make if one cake requires ½ kg of sugar?”
- An example of students' response using “finding a whole given a part” interpretation – “Misha picked 1 ¾ bucket of mushrooms. It is only a half of what Vanya picked. How many buckets of mushrooms was picked by Vanya?”
- An example of student’s response using “missing factor” meaning – “How many times 1 ¾ larger than ½?”
- As mentioned by one of our reviewers, some students in both groups attempted to mistakenly replace division scenario by multiplication problem - “(Since dividing by ½ is the same as multiplying by 2)… Misha has 1 ¾ liters of lemonade and Masha has it twice more. How much lemonade does Masha have?”

Results – Students (cont.)

- The student question 3 was identical to teacher question 6 and asked whether the given statement is ever true.
- Several types of students' responses emerged from the analysis of student work.
- Among them are incorrect responses such as "never true" and "impossible", partially correct responses such as "*true if a=b* and *c=d*" (or "*true if a=b=c=d=1*", and correct response – "*true if c=d*". Only one student in Irina's group was able to come up with a partially-correct solution whereas in Marina's group 8 students offered correct solutions and 2 – partially correct solutions.
- Examples of students' incorrect and correct response. Incorrect response – "It is never true because when you divide fractions you have to flip the second fraction". An example of students' correct response – "It could be true if *c/d = 1 or c=d*".

Thank you!