ZDM Mathematics Education (2017) 49:237–248
DOI 10.1007/s11858-016-0821-9

ORIGINAL ARTICLE

Preservice teachers' knowledge of interdisciplinary pedagogy: the case of elementary mathematics–science integrated lessons

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Accepted: 10 October 2016 / Published online: 13 October 2016

Abstract The purpose of the study is to explore how elementary preservice teachers' mathematics–science integrated teaching strategies changed as a result of participating in exemplary interdisciplinary activities with multiple themes across school curricula. The participating elementary preservice teachers (n = 28) were recruited for this study from the College of Education students enrolled at a medium-sized southwestern research university in the United States. A qualitative methodology with pre-and-post data collection from open-ended surveys was used in the current study to explore the development of preservice teachers' mathematics teaching strategies with connections to science themed activities before and after an 8-week intervention. In general, the results from the pre-and-post surveys revealed that the preservice teachers' interdisciplinary knowledge of using science-themed activities as instructional approaches for teaching mathematics had remarkable changes across all four science content areas including physics, chemistry, biology, and environmental and space science. This study provided additional empirical evidence on how contextualized mathematics educational activities, in the current case using the association between science and mathematics, can be used as effective teacher education resources for developing teachers' capacity for designing mathematics lessons.

Keywords Teacher education · Mathematics education · Interdisciplinary curriculum · Mathematics–science integration

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1 Introduction

Mathematics and science integration has a long history in modern education, and early publications can be traced back to the 1900s (Berlin and Lee 2005). Numerous studies examining science and mathematics integrated instruction in the United States, at both the national and state curriculum levels, have been conducted attempting to determine what precisely are the connections between the mathematics and science learning objectives (e.g., AAAS 1998; Cady and Rearden 2007; Frykholm and Glasson 2005; NCTM 2000; NSTA 2003). Across the world, educational researchers and curriculum theorists have also investigated methods, designs and classroom practices of science–mathematics integrated education. Examples include studies performed in Australia, (e.g., Hudson et al. 2014; Venville et al. 2012), Canada (e.g., Samson 2014), Ireland (e.g., Treacy and O'Donoghue 2014), Korea (e.g., Kim and Bolger, 2016), Germany (e.g., Blomhøj and Kjeldsen 2009), Singapore (Lam et al. 2013), Turkey (e.g., Saçkes et al. 2012). Collectively, these research studies have examined different methods for creating associations between mathematics and science to facilitate teaching mathematics and science content across K-12. However, there are as of yet no known replicable methods for assisting preservice teachers to develop instruction that effectively highlights the connections between mathematics and science. The goal of the current study was to help fill this research gap by examining how preservice teachers' pedagogical abilities at developing science-themed mathematics teaching strategies improved after they undertook a less difficult task, namely participate and evaluate arts-themed mathematics teaching strategies. The following research questions are addressed in the current study: (1) how did elementary preservice teachers' interdisciplinary pedagogy change as a result of participating in exemplary interdisciplinary mathematics activities? (2) How do such changes differ in teachers' interdisciplinary pedagogy for mathematics related to specific science contexts?

A variety of ways of supporting integration of mathematics and science with different models and principles were offered by researchers via the music–math integration model that was presented (e.g., Drake 1991; Davison et al. 1995; Fogarty 1991; Kiray 2012). For example, in Drake's framework (Drake 1991), the integration can be regarded in three ways: multidisciplinary, interdisciplinary, and transdisciplinary models are included. In Fogarty's framework (Fogarty 1991), the integration can be classified into 10 levels that may be characterized by fragmented, connected, nested, sequenced, shared, webbed, threaded, integrated, immersed, and networked models. Later on, Davison and his colleagues (1995) proposed another framework with five dimensions for analyzing science and mathematics integration including (1) discipline specific integration, (2) content integration, (3) process integration, (4) methodological integration, and thematic integration. More recently, Kiray (2012) introduced a balanced model of mathematics and science integration for equal value of content and standards, with five areas including content, skills, the teaching–learning process, affective characteristics, measurement and assessment. Similar to the diversity of integration models, mathematics and science educators also had different options for effective ways of teaching mathematics and science through interdisciplinary strategies. For example, Zemelman, Daniels, and Hyde (1993) advocated that problem solving should be the key process in mathematics and science integrated instructions, and they further itemized multiple sections to be evaluated, including the use of hands-on activities, group collaborations, discussion, estimations, technology, and so forth. Berlin and White (1994) offered another model for mathematics–science interdisciplinary teaching with six aspects for assessing the quality of integrated lessons, including learning methods, comprehension approaches, thinking skills, knowledge of content, students' dispositions, and teaching strategies.

A general finding across this research is that there exist excessive challenges that are blocking teachers' implementation of mathematics and science integrated methods, and most lessons were developed based on a superficial level of integration that is lacking the conceptual rationale necessary for identifying and designing interdisciplinary topics (Samson 2014). However, compared with the large number of studies focused on theory development, only a limited number of studies paid attention to the use of a mathematics and science integrated approach in teacher education (Sriraman and Knott 2009). Existing empirical literature on integrated science–mathematics education in teacher education is mainly focused on assessing preservice teachers' beliefs or perceptions, and more studies are needed that investigate the effects of such interdisciplinary experiences on preservice teachers' interdisciplinary pedagogical content knowledge, specially their instructional design capacities (Lam et al. 2013).

2 Interdisciplinary pedagogical content knowledge: a theoretical perspective

Their teachers' pedagogical ability has been repeatedly identified as the essential factor for impacting students' learning outcomes and motivational levels (Baumert et al. 2010; Hattie 2009). Highly qualified teachers, especially mathematics teachers, are—by definition—those teachers that have substantial knowledge about varying teaching approaches relevant to their subject areas. This general knowledge base includes the following: classroom management, lesson design and implementation; understanding of the mathematics content; and methods of effectively engaging students to access these mathematical ideas. These types of knowledge were theorized by Shulman (1987) as pedagogical knowledge (PK), content knowledge (CK) and pedagogical content knowledge (PCK) respectively. PCK is a general skillset that collectively focuses on the overlap part between the PK and CK—specifically, how individual topics of subject knowledge are systematized, modified, and exemplified for classroom teaching. Several mathematics education researchers (e.g. Ball, Thames, and Phelps 2008; Park and Oliver 2008) further specified two core dimensions in PCK, which differentiated teachers' knowledge between (1) students' subject-specific conceptions together with misconceptions, and (2) subject-specific instructional strategies and representations.

While the concepts related to PCK have been extensively explored, discussed and revised by numerous researchers (e.g. Ball et al. 2008; Borko and Putnam 1996; Cochran et al. 1991; Mishra and Koehler 2006; Magnusson et al. 1999) in the past three decades, especially in the field of mathematics, there has been both empirical investigation and theoretical development focused on improving teachers' PCK across disciplines such as mathematics and science. The PCK based on interactions among multiple subjects is entitled interdisciplinary pedagogical content knowledge (IPCK) in this study. By extending Shulman's (1986, 1987) model of PCK and Frykholm and Glasson (2005)'s concept of pedagogical context knowledge, the IPCK framework is specified as an explicit knowledge of interdisciplinary pedagogy (see Fig. 1). As Fig. 1 shows, the framework has three essential categories of knowledge: pedagogical knowledge (PK), content knowledge in subject A (CK-A), and content knowledge in subject B (CK-B). The current theoretical framework demonstrates that

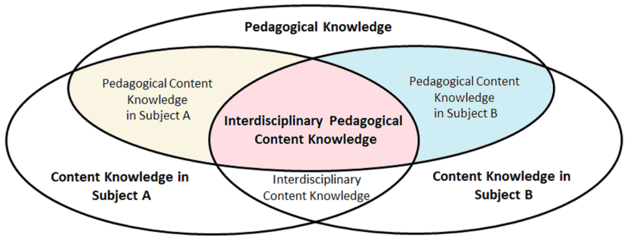

Fig. 1 Graphic representation of interdisciplinary pedagogical content knowledge (IPCK)

overlapping these three categories of knowledge will result in the emergence of four additional types of knowledge, namely pedagogical content knowledge in subject A and subject B (PCK-A and PCK-B), interdisciplinary content knowledge (ICK), and interdisciplinary pedagogical content knowledge (IPCK).

In brief, interdisciplinary pedagogical content knowledge (IPCK) is the specific capacity for teachers to accomplish the following: (1) work with interdisciplinary considerations that include an understanding of the representation of concepts using themes across curriculum boundaries; (2) apply pedagogical methods and interdisciplinary themed activities in addressing content areas from multiple subjects simultaneously; (3) identify knowledge connections within and between particular subjects, and develop lessons based on such connections; and (4) employ knowledge of how interdisciplinary explorations can be developed as a part of an instructional process wherein students link existing knowledge across curricula, while presenting that new knowledge through contexts from multiple subjects. Previous research has indicated that the development of teachers' understanding of teaching, content, curriculum, pedagogy, and students, especially in the subject of mathematics, are all needed to transverse the subject area boundaries that divide the individual disciplines, and this requires development of teachers' interdisciplinary pedagogical content knowledge (An and Tillman 2014; An, Tillman, Shaheen, and Boren 2014; An, Tillman, and Paez 2015; An, Tillman, Zhang, Robertson, and Tinajero 2016) (see Fig. 2).

3 Methodology

3.1 Research setting and participants

The location of the current study was in a research university located within a bilingual border metropolitan area in the southwestern part of the United States. The university has an enrollment of over 20,000 students, and more than three quarter of the student body are Hispanic. The current study was part of a larger research project focused upon developing preservice teachers' interdisciplinary pedagogical content knowledge. The aggregate participants in the larger research projects were 342 preservice teachers from six cohorts who were enrolled in either the elementary generalist certificate program, the elementary bilingual generalist certificate program, or the special education certificate program, from spring 2013 to fall 2015. For the current study, data were collected from 28 preservice teachers who had enrolled in the fifth cohort. Specifically, data were employed from participants who had registered in one of three sessions of a senior level mathematics teaching methods course. Among the participants, 26 out of 28 were female preservice teachers and 23 of them were self-reported as Hispanic. All participants finished the corresponding science teaching methods course before participating in this study. In particular, they systematically studied science content concepts associated with the applicable Texas Essential Knowledge and Skills (TEKS) and National Science Education Standards (NSES), as well as designing and implementing investigations for elementary students using scientific inquiry.

3.2 Research design and intervention

A qualitative methodology with pre-and-post data collection from open-ended surveys was used in the current study to explore the development of preservice teachers' mathematics teaching strategies with connections to science themed activities before and after an 8-week intervention. The data collection and the intervention in the current study occurred during one regular semester over a period of 13 weeks. Specifically, the pre-surveys were assigned in week one and two and then an 8-week intervention was provided through weeks 3–11; finally the post-surveys were assigned in weeks 12 and 13. The intervention process is described in the following paragraphs, and the data collection procedure is specified in the next section.

The intervention constituted of a series of eight sets of exemplary interdisciplinary activities, primarily with arts–mathematics connections (e.g. composition/music playing, choreography/dancing, and visual arts themed mathematics pedagogy), which were selected from 509 lesson plans designed by preservice teachers in the previous three cohorts who participated in our preliminary research of interdisciplinary lesson plan design and evaluation (see An and Tillman 2014; An et al. 2016; Tillman, An, and Boren 2015). The rationale of offering mathematics–arts integrated pedagogy as interventions for developing preservice teachers' mathematics–science integrated pedagogy is the commonalities among the ways that artists, mathematicians, and scientists alike use analysis and synthesis to create elegant solutions to complex challenges. The creative process connecting mathematics–arts and

Fig. 2 Sample pedagogical links between science and mathematics topics proposed by participants during the discussion

mathematics–science both involve a combination of analysis—such as asking how something works, often using scientific or behavioral theories in the explanation—and synthesis—such as asking what form we should give to something in order to serve a particular purpose. Sitting between analysis and synthesis is the concept of *design*, where mathematical reasoning is often an important key in finding optimal solutions. This study examines the potential for an integrated mathematics–arts teaching approach as a bridge to enable preservice teachers' pedagogy transfer from the arts to non-arts content (such as science) by developing their interdisciplinary teaching expertise within an easier domain, namely arts–mathematics, before transitioning to a more difficult domain, namely science–mathematics, in order to provide essential scaffolding. For space limitation reasons, the examples of arts-themed activities will not be detailed in this paper (interested readers can find detailed examples in the articles cited above) so that the science-themed activities can be highlighted.

Although arts–mathematics links were presented (in 6 out of 8 intervention sessions) as the major exemplary interdisciplinary pedagogy for preserivice teachers to understand strategies of using cross-disciplinary materials to develop mathematics exploration activities, two sessions were offered with the emphasis on science themed mathematics pedagogy in the intervention. For example, in one of the sessions emphasizing science themed mathematics pedagogy, three video lessons taught by astronauts in the space shuttle during their missions orbiting earth in outer space were presented. Among the three lessons, one lesson was taught by Chinese astronauts and two lessons were taught by American astronauts. The astronaut-teachers covered over forty topics in science, and a series of experiments were conducted in the microgravity environment of outer space (see An et al. 2016, for details about the lesson topics and structures). The leading teachers in both countries were female astronauts—the Chinese lesson was taught by Yaping Wang in the space shuttle Shenzhou during the mission Shenzhou 10, and the American lesson was taught by Barbara Morgan in the space shuttle Endeavour during the mission STS-118. After watching the video lessons, the participants had group discussions about possible pedagogical links between mathematics and science themes covers by the three space lessons. Participants proposed that the science content in these three lessons could be used as resources to develop mathematics lessons with topics including algebra (e.g., ratio, proportion, function and pattern), number and operation (e.g., percentage), measurement (e.g., time, volume, distance, temperate, angle and area), data analysis and probability (e.g., permutation, frequency, and normal distribution) and geometry (e.g., location and transformation).

3.3 Instrument, data collection and analysis

A pretest–posttest data collection design was utilized to assess the effects of the intervention on participants' science contextualized mathematics teaching strategies. Open-ended surveys assessing science themed mathematics teaching strategies were developed based on a previously validated instrument (see An et al. 2015) for assessing preservice teachers' interdisciplinary pedagogical knowledge in teaching mathematics. The purpose of this open-ended questions survey was for preservice teacher participants to specify their use of science activities as a pedagogical strategy for facilitating students' understanding of mathematical concepts, as well as their perceptions about the pedagogical connections between music and mathematics. Specifically, the survey was designed with four sections, each with four individual tasks, during which the participants designed science activities that employed mathematics pedagogy. The four science content areas included were: (a) physics, (b) chemistry, (c) biology, and (d) environmental and space science. Due to space limitation, the environmental and space science part was excluded in the current report, and these data will be reported in future articles.

To minimize the testing fatigue from intensive writing, both the pre-survey and post-survey were assigned in two parts, and were completed during regular class meeting times. Two questions in each survey asked students about (1) what are connections between elementary mathematics and a specific content area in science, and (2) their plans to design and implement a mathematics lesson with science contextualized activities in their future classroom. Each survey was scheduled about 30 min for participants to complete, and the average number of written words among all participants on the pre-survey was 252, and on the post-survey was 407. The pre-survey part I and part II were provided in week one and week two respectively, and the same two-part surveys were provided in week 12 and week 13 respectively as the post-survey. In survey part I, participants were requested to provide their instructional strategies and beliefs about pedagogical connections between mathematics and physics, as well as mathematics and chemistry. The content areas that were the focus in part II of the survey were mathematics and biology, as well as mathematics and environmental and space science.

Grounded theory (Corbin and Strauss 2008) was employed to identify and compare preservice teachers' strategies and methods used to integrate mathematics pedagogy with science themed activities between pre-surveys and post-surveys. In general, all the collected participants' responses were coded and examined with a two-tiered process: (1) a macro-dimension analysis focused on mathematics and science content interactions, and (2) a micro-dimension analysis focused on specific mathematics and science learning processes within concrete scenarios. Specifically, written responses between pre-survey and post-survey were compared instance to instance while developing tentative categories by identifying similar and dissimilar science themed mathematics teaching approaches offered by the preservice teachers. Once the tentative category was saturated, all the other written responses were coded along the same categories. Next, the second round of category development was carried out in order to refine tentative categories based upon emergent grouping themes. To reduce bias in coding, the qualitative data were analyzed by three different researchers, who each iteratively coded the data independently. With an inter-rater agreement rate of 88 %, any conflicts during the coding results were resolved collectively through face to face negotiations among the three researchers.

4 Results

In general, the results from the pre-surveys and post-surveys revealed that the preservice teachers' interdisciplinary knowledge of using science-themed activities as instructional approaches for teaching mathematics had remarkable changes across all three science content areas examined (including physics, chemistry, and biology). Compared with the pre-survey responses, preservice teachers in the post-surveys proposed a noteworthy number of interdisciplinary pedagogical approaches based on sophisticated mathematics–science connections. In general, preservice teacher participants proposed in the pre-surveys a total of 19 different methods for using science activities in mathematics lessons, and the number of methods increased to 55 in the post-surveys. Among the three content areas in science, biology was the one that had the most changes in regard to developing mathematics pedagogy, and physics was the one that impacted the least amount of change.

In addition to the identified differences in number of teaching methods, another key difference between the pre-surveys and post-surveys was the sophistication level of teaching methods. While assessing the level of teaching methods, special attention was paid to whether the interdisciplinary pedagogy between science and mathematics went beyond being a cover-story with merely background purposes, but instead: (a) provided problem-based/inquiry-based activities for students to construct knowledge, (b) represented mathematics concepts with multiple forms including visually, with words, with symbols, and with numbers, as well as other types of dynamic forms; and (c) allowed students to solve mathematics problems by using a variety of problem-solving strategies that lead to a diversity of legitimate answers. Each of these three evaluation aspects were emphasized in the example activities presented for preservice teacher participants during the intervention, and the evaluation of interdisciplinary pedagogy in the pre and post surveys was closely related to the intervention, with the only difference being that the participants transferred the contextualization of mathematics teaching from an arts-context to a science-context. Overall, in the pre-survey most of the approaches the preservice teachers offered were based on superficial associations between mathematics and science—topics in mathematics were primarily limited to teaching the concepts of measurement such as time, temperature, weights, dimensions of physical items, and liquids within science contexts. In contrast, in the post-surveys a majority of the approaches that the preservice teachers proposed were based on more sophisticated science activities that involved inquiry learning processes across all four science content areas (see Tables 1, 2, 3).

Specifically, in terms of contextualizing science activities within physics, in the pretest 26 (93 %) participants proposed seven different plans for teaching mathematics; in the posttest, eight more new music themed strategies were identified and all 28 (100 %) participants proposed at least one way to teach one of the mathematics concepts by connecting it with physics. In terms of the content area of chemistry, in the pretest, 19 (68 %) participants proposed five different strategies to teach mathematics; in the posttest seven more new chemistry themed strategies were identified and 27 (96 %) participants proposed at least one way to teach one of the mathematics concepts by connecting it with chemistry. In terms of the content area of biology, in the pretest, 17 (61 %) participants proposed four different strategies to teach mathematics; in the posttest 11 more new biology themed strategies were identified and 25 (89 %) participants proposed at least one way to teach one of the mathematics concepts connected with biology. As illustrative cases, the instructional design of science themed mathematics lessons by three preservice teachers, James, Jessica, and Alexandra (names have been changed to pseudonyms), are presented as examples of the four individual science content areas that were explored, as well as the differences between the instructional designs that the participants proposed in the pre-surveys and the post-surveys. The cases were selected according to the rationale that these participants: (1) made noteworthy changes, and (2) demonstrated typical ability among the participants regarding their instructional designs between the pre-survey and the post-survey.

4.1 Physics themed mathematics lessons—the case of James

One of the preservice teachers who participated in this study, James, proposed a strategy of teaching measurement connected with buoyancy in physics. The interdisciplinary pedagogy for mathematics was developed based on Archimedes' principle; however, he failed to offer further exploration for students to discover the pattern between buoyancy force and the submerged volume times. The concept of mathematics in the activity was limited to finding volume using a graduated cylinder, and no formulas of volumes and other concepts related to three dimensional geometry were presented. James described his instructional design as follows:

Table 1 Comparison of mathematics-physics interdisciplinary teaching strategies that emerged between pre-surveys and post-surveys

Major themes	Major physics themed mathematics teaching strategies
Strategies emerged in pre-surveys	Quantify what colors absorb the most heat by place different colored papers outside with thermometers Put different type of liquids (e.g., rubbing alcohol, vegetable oil, milk, dish soap, maple syrup) into a clear container and observe the pattern of density from layers Measure and compare properties of a series of objects with different size (e.g., length) and mass (e.g., weight) Drop different items (e.g., basketball and tennis ball) at the fixed point and observe whether they reach the floor at the same time and measure the falling time Push toy cars across a surface, measure the time of motion and distance and then calculate the average speed Place different objects into a graduated cylinder and observe the differences of water levels Put different food items (e.g., chocolate, butter, and cheese) into a heated cupcake pan and create a graph to record the time of melting from solid to liquid
Strategies emerged in post-surveys	Create levers or push and pull systems and then compare the efficiency through experiments Build ramps for toy cars with different heights and slope, and then explore the variables that impact the speed Create a maze based on pool noodles and compare the time of falling down process for different objects Build bridges with popsicle sticks in different shapes and explore the most support to provide from collapsing Place the compass against the mirror and shine the light or laser into it then identify the angle and reflection Prepare a number of beakers filled with different amounts of water, measure and compare the rate of temperature changes by using a thermometer Drop a number of objects with different shape, sizes and weight into a box with sand, and explore which kinds of objects make the biggest indentation in the sand Explore the concept of dry friction by rolling balls with different sizes and materials from the same point, and compare the traveling distance

Table 2 Comparison of mathematics-chemistry interdisciplinary teaching strategies that emerged between pre-surveys and post-surveys

Major themes	Major chemistry themed mathematics teaching strategies
Strategies emerged in pre-surveys	Find atomic mass or neutron of an element by solving the equation with missing variables Introduce units and tools for measuring small amounts of chemicals including solid, liquid and gas Use pH Test Strips to test a variety of liquids and compare pH levels and determine the acid or base Combine salt with water and measure the weights of the salt before and after evaporation Measure the differences of spin speed between raw eggs and boiled eggs to explore the chemical change of molecular structures
Strategies emerged in post-surveys	Have the children create molecules by grouping atoms and electrons and explore mathematical patterns Discover elements of hydrogen and measure the atomic number, atomic mass, and all the other characteristics Mix certain quantities of substance together and then determine how the reaction can change by manipulating the number of items Use the "picket fence" methods to organize numbers and information for chemistry and other contexts Explore 3-D geometry through the creation of molecules in each compound by using jelly beans and toothpicks Experiment with different materials to find out which mixture makes the best crystals, then explore shapes and symmetry Explore algebraic patterns in atoms and determining how many electrons one atom has to lose or share during bonding processes

In the area of physics, students are able to explore and collect data from the world around them. With the use of mathematics students learn to record and calculate data while using this information to develop logical predictions and hypothesis. One math activity that can be done through physics is calculating the volume of different objects using the displacement method. Students will place three different objects inside of 2 l of water using a graduated cylinder. They will then measure the amount of water before and after the objects are placed inside. They will be able to notice that the water level has now increased and will compare the two values. After subtracting the initial water volume from the second, they will conclude the volume of the object is equal to the amount of water displaced. These activities allow students to participate in experimental methods and practice their problem solving skills using mathematical processes.

Different from the linking science and mathematics without opportunities for students to practice higher level of thinking in James's instructional design in the pre-survey, his revised lesson offered a more substantial degree of integration, with multiple connections between the physics theme and mathematics content, by utilizing physics to illustrate algebraic patterns at a more sophisticated level.

Table 3 Comparison of mathematics-biology interdisciplinary teaching strategies that emerged between pre-surveys and post-surveys

Major themes	Biology themed mathematics teaching strategies
Strategies emerged in pre-surveys	Plant beans in a clear cup with soil and observe how many days it took for the bean to sprout and to days it took for the stem to come of the soil
	Plant beans with different nutation and conditions, control variables and compare the growth by measuring the height of the plants
	Explore human body by counting the number of bones and teeth, labeling the nerve system and calculate the percentage of body fat among different people
	Have students run around and do physical activities while measuring their heart rate, and creating charts of the difference between heart rates
Strategies emerged in post-surveys	Identify different ant farms and observe how the ants work to make there colony, and collect data on the different groups to investigate the optimal number of ants to create a colony
	Put different types of bread in a zip lock bag and observe the food rotting process, control variables to compare the rotting speed and use mathematics scales to estimate the amount of mold
	Use Punnett squares to represent the cross breeding of two organisms within a species, and create a "blood/crime scene" problem for students to use mathematical reasoning to determine the criminals
	Extract DNA sample from one of fruits and view the strands of DNA under a microscope and investigate the geometrical structures and mathematical sequences
	Identify the probability of a person or plant's offspring getting a recessive gene in different cases of dominant and recessive gene combinations
	Explore the concept of symmetry, congruent, and similar as well as explore Fibonacci patterns across different type of plants and insects
	Identify patterns in life circle across different insects and plants by numbering the sequence stages, and investigate mathematical structure in different type of food chains
	Explore mathematical structure of extended families, and identify patterns of genetics throughout a number of generations
	Have students gather up leaves and sort the leaves out in based on different ways of categories and construct statistical graphs and Venn diagram to represent their findings
	Have students create habitats and facilitate them to explore and recreate the features of various habitats while using measurements and scale
	Find the differences in height and weight of different kinds of animals and explore the relationship between the size of the animal versus the amount of food and water they need to consume per day

Rather than passively recording data by using a measurement instrument, James's revised lesson connected the algebraic concepts of a fixed ratio between height and speed, as well as an irrelevant variable of slope. In this lesson, students would be given the opportunity to discover the equation by themselves from multiple physics experiments. James provided the following description of the lesson he designed:

> One specific physics activity that I thought of for elementary students would be to have them engage in an activity where they roll a car down a ramp (inclined plane). There are several mathematics concepts within this activity. One is that they would have to weigh the mass of the car, since the formula for force is mass multiplied by acceleration. Another concept would be measuring the height of the inclined plane. I would require the students to do several trials with the plane at least three different heights. Students can add books to make the plane more or less inclined to their liking. Here, they then would need to take a ruler or meter stick to measure the height as well as the base of the inclined plane. If one ramp is to be more steep while the other is to be less steep when we are to release the balls at the same time, the two balls will reach the finish line at a different time. Why is that? These questions should encourage students to think of possible solutions, and meanwhile we can introduce them to the mathematical equation that explains it.

4.2 Chemistry themed mathematics lessons—the case of Jessica

Jessica introduced a mathematics lesson with water evaporation activity as the contextualization strategy for teaching measurement, data collection and analysis. However, this water evaporation activity allowed students to measure the weight of salt for limited cases, and the quantitative analysis process was limited only to comparing decimal numbers. No follow up investigations and discussions of mathematical concepts were included in the instructional design. Jessica provided the following description of the instructional process in her lesson plan:

> Because chemistry deals with matter, and the changes that occur in that matter, it is necessary to evaluate and measure how matter changes and why changes have occurred. One example of a chemistry lesson that teaches mathematics includes combining salt

> with water in order to discover that even though the salt seems to have disappeared, it is still there because matter is neither created nor destroyed even though it may undergo a change. With this particular chemistry lesson, students would be required to measure a certain amount of water and a certain amount of salt. Additionally, the students would need to find the weight of the salt by first measuring the container and then the container with the salt. In order to calculate the weight of the salt, the students would need to find the difference of the weight of the container with salt and the weight of the container alone. Next, the students would make predictions about what will happen to the salt when the water evaporates. When the water does finally evaporate, the students will compare their predictions, initial observations, and weights of the salt before and after evaporation. During this particular chemistry lesson, basic mathematical skills are necessary to be successful in the completion of this experiment. Therefore, it is clear that chemistry relies heavily on many mathematical skills in order for the lessons to be effective.

Still using salt and water as key materials for students to conduct chimerical experiments and explore mathematics, Jessica in the post-survey improved her teaching strategies by added a variety of open-ended thinking tasks for students to make sense of geometry in an innovative way. Her lesson reported in the post-survey taught the concept of symmetry based on the different types of snowflakes created from different chemicals. Students would be given the chance to explore symmetrical lines in the snowflakes they made by themselves. Jessica stated the following objectives in her lesson plan:

> My activity for elementary [students] can be to have them experiment with different materials to find out which mixture makes the best crystals. The materials required for this activity are containers, water, qtips, black construction paper, and 4 substances to mix with the water (alum, Epsom salt, light salt, and sugar). As a class we will discuss the components of each of these 4 substances, and then the students will predict which mixture will make the best crystals. Have the students label each container that will correspond to their mixtures and then they will mix the water and the substances, measuring the water and substances to make sure that they use the same amount of water for each container. After this the students will fold their black construction paper into 4 sections, label them to correlate to the mixture, and then they will be asked to take a qtip and draw a symmetrical snowflake (one snowflake for each mixture). As the pictures dry have the students describe on a separate sheet of paper how their snowflakes look like. They must state the shapes they used and how many lines of symmetry each of their snowflakes are made up of. After the pictures dry, students will examine and determine which mixture made the best snowflakes, and then we will later discuss how snowflakes form and how they are made up of crystals.

4.3 Biology themed mathematics lessons—the case of Alexandra

Among the 13 preservice teachers who proposed a mathematics–biology interdisciplinary lesson based on plant growing and measurement activities in the pre-survey, Alexandra is one who designed a lesson with this plausible activity. In her multiple-week lesson plan the students are required to bring their own plants to the classroom, record the plant's growth information and finally make a line graph. Although Alexandra thought students may have a positive attitude in exploring mathematics in their own plants, this kind of biology themed lesson offered only a cover-story for mathematics and the students' had limited chance to understand mathematics in multiple approaches. For example, Alexandra provided the following instructional design for her students:

> I think a lesson that would effectively integrate biology and math would be to have the students grow their own plants in the classroom. This activity would take place over the course of several weeks, and the students will record the progress of the plants. They would measure the growth of the plants and how long it takes them to grow, the students would also keep track of all their data. By the time the students have recorded the height of their plant, they would then create a line graph of the height measurements they recorded of their plant. I think this activity will make the class a more welcoming environment as they bring in their own plant.

In the post-survey, Alexandra redesigned a mathematics lesson focused on the topic of probability based on the concept of genetics. In contrast to her original lesson which had repetitive measurement tasks on the same plant, in her updated lesson plan in the post-survey she offered a profound connection between mathematics and biology through hands-on activities. Specifically, she used jelly beans with different colors as the key manipulatives for illustrating how genetics works through three generations, and the results of the biology experiment also demonstrated probability concepts. As she described:

> My ideas for the math lesson involves the students learning how genetics works. To begin, the students

will get into groups and be given 7 small cups and jelly beans (red, green, blue, and yellow). They will label the cups as Grandfather 1, Grandfather 2, Grandmother 1, Grandmother 2, Mother, Father, and Child. Next they will place 8 jellybeans (making sure there are 2 of each color) inside each of the cups labeled Grandfather 1 and 2 and Grandmother 1 and 2. Then the students will randomly select 4 from the first set of grandparents to create the Mother and then do the same for the second set of grandparents to create the Father. The students will repeat the same process from the Mother and Father to create the Child. Once they are done, the students will return the jelly beans to the grandparents and repeat the process 3 more times and keep a data table of the results. The students will be able to see that there are many possible outcomes for the child.

5 Discussion

The overall findings from both pre-surveys and post-surveys indicated that in the current study the participating preservice teachers' interdisciplinary pedagogical content knowledge was positively impacted by the intervention of participating and evaluating exemplary mathematics activities focused on different pedagogical connections across school subjects. This study provides additional empirical evidence about how contextualized mathematics educational activities, which in the current case explored the association between science and mathematics, can be used as effective teacher education resources for developing teachers' capacity to design innovative mathematics lessons. Compared with the instructional designs that preservice teachers' proposed in the pre-surveys, lessons in the post-surveys displayed more new teaching methods with more profound interdisciplinary connections. Although the preservice teachers in the pre-survey proposed a number of mathematics lessons that linked with all four major science content areas, prior to the intervention there were only a fairly limited number of pedagogical methods presented that went beyond using science activities as a means for conducting basic measurement. The findings also illustrated that some content in science, such as biology, is more difficult than other content, such as physics, for preservice teachers in developing mathematics pedagogy, as reflected in the pre-survey.

In the post-survey, however, the findings revealed that the preservice teacher participants demonstrated a wider diversity of science-themed activities integrated into mathematics lessons, and they also covered more comprehensive mathematics–science connections at the subject level through inquiry-oriented processes. Regarding science activities for teaching mathematics, a variety of scientific topics were all used as interdisciplinary themes for contextualizing mathematical concepts. As the cases of James, Jessica, and Alexandra's instructional designs illustrated, rather than merely using science related tasks as warm-up practice during mathematics lessons such as in the pre-surveys, during the post-surveys a range of pedagogical approaches supporting mathematics content areas, such as algebra, geometry, data analysis and probability, were reported. These new teaching approaches included (a) exploring ratios connecting physics and algebra; (b) visualizing symmetry, connecting chemistry and transformations in geometry; and (c) conducting experiments with heredity that demonstrate connections between biology and probability.

Researchers across the globe, in both centralized educational systems (e.g., Kim and Bolger 2016; Lam et al. 2013; Saçkes et al. 2012) and de-centralized educational systems (Venville et al. 2012; Samson 2014; Cady and Rearden 2007) have identified, implemented, and assessed various integrated methods for teaching mathematics and science content; however there has not been a definitive answer as to how best to improve preservice teachers' instructional design for interdisciplinary mathematics–science pedagogy. The current study attempted to help fill this research gap by examining potential techniques and prospects for facilitating preservice teachers, development of their own knowledge for supporting science-themed mathematics pedagogy, by first employing a less intimidating form of interdisciplinary mathematics education, namely arts-themed mathematics education. The improvement of instructional designs proposed by the preservice teachers from the pre-surveys to the post-surveys, can be reasonably attributed to the intervention, and the effects of the intervention were partially displayed by comparing the results from the pre-surveys and post-surveys. The selected exemplary interdisciplinary mathematics lesson designs presented during the intervention facilitated the preservice teachers' comprehension of the potential for employing innovative techniques when preparing lessons that introduce mathematics concepts. Specifically, in the present study the analysis of the data collected showed that preservice teachers' development of interdisciplinary pedagogical content knowledge is viable, as most participants learned how to apply activities from science curriculum meaningfully to design lessons that have the potential to help students conceptualize abstract mathematics concepts during inquiry-based knowledge construction processes.

Teachers' effectiveness in designing and implementing lessons has been recognized as a crucial factor in achieving the goal of equity in education—however, the mathematics achievement gap between minority and non-minority students is still evident (Darling-Hammond and Baratz-Snowden 2007). To provide sufficient preparation for teachers before they start classroom teaching, to ensure their pedagogical and content knowledge, one of the key missions for teacher education programs is to better understand students' patterns of mathematics learning and their corresponding misconceptions (Swars et al. 2006). However, evidence across studies displayed that a majority of elementary preservice teachers are not sufficiently prepared (Ball 1990; Darling-Hammond and Baratz-Snowden 2007; Fuller 1997) and teaching outcomes from less competent peers are significantly lower than from highly qualified teachers (Singham 2003). In particular, Labaree (2008) noted that traditional teacher education programs are customarily organized to emphasize mono-disciplinary instructional methods, which highlight distinct disciplinary boundaries. Such teacher education programs that are rooted in separating the mathematics and science disciplines into discrete silos have failed to offer relevant and engaging pedagogical practices for preservice teachers to develop the style of mathematics pedagogy needed to reach out successfully to the numerous demographics of historically underserved students. Designing and implementing effective mathematics teaching strategies to engage students so that they participate in mathematics discussions and evolve their own conceptual understanding in meaningful mathematics activities is essential (Gresham 2008).

Based upon the findings from this study and other related research, teacher preparation programs should attempt to support preservice teachers in experiencing, evaluating and understanding how to develop authentically contextualized mathematics instruction that avoids using trivial adornments with only entertainment purposes during mathematics lessons. Instead, competent teachers should be able to identify, choose and adapt interdisciplinary exploration opportunities which enable structured tasks wherein students apply high-level thinking processes (e.g., analysis, synthesis, and evaluation) during their daily lessons (Drake and Burns 2004). This purpose can be accomplished by having teacher educators prepare future teachers via interdisciplinary orientations that cross different subject areas and thereby help students achieve authentic understanding and interest in the subjects taught (Knoblauch and Hoy 2008). Results from the current study and our previous studies (An et al. 2016; An and Tillman 2014) suggest that cross-disciplinary approaches that employ appropriate connections between mathematics and the other school subjects, such as science, deserve additional attention in teacher education programs.

Before proceeding to summative conclusions, a few limitations to the present research study should be clarified. First, the sample size was fairly small in the current study and most participants were from a single demographic group (i.e., females with Hispanic ethnicity), and therefore the effects of a similar intervention are still unknown for different gendered or ethnic demographics. Additionally, in the current study's intervention we connected multiple topics across arts, science and mathematics, and the complexity in the resulting pedagogical structure could be problematic both logistically and during research comparisons. The findings from this study invite additional empirical investigation of how to improve teacher education programs by adding more interdisciplinary components, as well as inviting more research to assess more thoroughly the effects of interventions with interdisciplinary themes. There is hence an opportunity for research examining many such potentially creative contexts, such as those that integrate science, social studies, the arts, and language arts, into the development of mathematics preservice teachers' pedagogical practices.

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