Department of Mathematical Sciences

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Linear Hybrid-Variable Methods for Advection Equations

The advection equation is the simplest partial differential equation (PDE) that allows an analytical solution; however, researches in numerical approaches to solve these equations have been active since decades ago, because they often shed light on how we can solve the more general hyperbolic conservation laws numerically. In this work, we propose a new hybrid-variable (HV) framework for advection equations by approximating both cell averages and nodal variables, and investigate the general existence, accuracy, and stability theorems of these methods. Several tools from other branches of mathematics are used in the analysis of HV methods. In particular, we use the Hermite interpolation theory to explicitly construct HV methods of any order accuracy, and use results from matrix analysis to prove the superior accuracy of HV methods comparing to classical finite difference schemes as well as the linear stability of sample HV schemes. In this regard, we generalize the upwind condition of classical finite difference methods to the context of HV discretizations. Further analysis includes the extension of Iserles's order star theory to branched Riemann surfaces to show that the stencil cannot be biased too much in the upwind direction. Lastly, we demonstrate how the methodology can be extended to higher dimensions and nonlinear systems. Numerical performance is assessed by various linear and nonlinear problems in one and two space dimensions.