Mathematical Sciences Fall 2024 Colloquium Series

Dr. Luis Valdez-Sánchez
University of Texas at El Paso (UTEP)

Friday, September 27 at 3:00pm
Bell Hall 143 and online via Zoom
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The Algebra and Topology of 3-Dimensional Genus Two Handlebodies

ABSTRACT: A genus $g \geq 0$ handlebody is a 3-manifold obtained by attaching g disjoint 1-handles to a 3-ball. Thus a 3-ball, a solid 'donut' (torus) and a double solid 'donut' are genus 0, 1, and 2 handlebodies, respectively.

Solid 'donuts' (tori) and their torus boundary play special roles in the classification of links in the 3-sphere $\mathbb{S}^3$ and 3-manifolds in general. This talk is about how, to a lesser but relevant extent, genus two handlebodies also play a crucial role in such classification problems. This is largely due to the fact that the fundamental group $\pi_1(H)$ of a genus two handlebody H, a free group of rank 2, is the only free group of rank ≥ 2 where an algorithm to decide if, for instance, a word is primitive (ie, part of a free basis), exists.

We start with the classification of mutually disjoint and nonparallel circles in the boundary of a genus two handlebody H, each of which represents a primitive word in $\pi_1(H)$, and show how it can be used to provide a classification of all 3-component hyperbolic links in the 3-sphere whose exteriors contain a spanning planar surface with large boundary slopes. These two classifications provide more support to a ~1995 conjecture by Miyazaki and Motegi, regarding Seifert fiber surgeries on hyperbolic knots in $\mathbb{S}^3$.

Hosted by: Dr. Emil Schwab

For further information, please contact Dr. Emil Schwab, eschwab@utep.edu