Factors and Zeros of Polynomials
Let \( p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \) be a polynomial. If \( p(a) = 0 \), then \( a \) is a zero of the polynomial and a solution of the equation \( p(x) = 0 \). Furthermore, \( (x - a) \) is a factor of the polynomial.

Fundamental Theorem of Algebra
An \( n \)th degree polynomial has \( n \) (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula
If \( p(x) = ax^2 + bx + c \), and \( 0 \leq b^2 - 4ac \), then the real zeros of \( p \) are \( x = \left( -b \pm \sqrt{b^2 - 4ac} \right) / 2a \).

Special Factors
\[
\begin{align*}
x^2 - a^2 &= (x - a)(x + a) \\
x^3 - a^3 &= (x - a)(x^2 + ax + a^2) \\
x^4 - a^4 &= (x^2 - a^2)(x^2 + a^2)
\end{align*}
\]

Binomial Theorem
\[
\begin{align*}
(x + y)^2 &= x^2 + 2xy + y^2 \\
(x + y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 \\
(x + y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \\
(x + y)^n &= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k \\
(x - y)^n &= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} (-y)^k
\end{align*}
\]

Rational Zero Theorem
If \( p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \) has integer coefficients, then every rational zero of \( p \) is of the form \( x = r/s \), where \( r \) is a factor of \( a_0 \) and \( s \) is a factor of \( a_n \).

Factoring by Grouping
\[
acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)
\]

Arithmetic Operations
\[
\begin{align*}
ab + ac &= a(b + c) \\
\frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} \\
\frac{a}{b} &= \frac{c}{d} \\
\frac{a}{b} + \frac{c}{d} &= a \frac{d}{c}
\end{align*}
\]

Exponents and Radicals
\[
\begin{align*}
a^2 &= 1, \quad a \neq 0 \\
(ab)^x &= a^x b^x \\
\sqrt[n]{a^m} &= a^{m/n} \\
\sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\
\sqrt[n]{a} &= a^{1/n} \\
\sqrt[n]{a^m} &= a^{m/n} \\
\sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}}
\end{align*}
\]

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