



**SOUTHWEST LOCAL ALGEBRA MEETING 2019  
THE UNIVERSITY OF TEXAS AT EL PASO**

The talks take place in room 309 in the Business Administration Building and the poster sessions are held in rooms 331 and 332.

PROGRAM

Saturday	23 February	Sunday	24 February
12:30–1:00 pm	Check-in and coffee/snacks	8:00–8:30 am	Continental breakfast
1:00–2:00 pm	Poster session	8:30–9:30 am	Anton Dochtermann
2:00–3:00 pm	Ezra Miller	9:30–10:30 am	Poster session
3:00–4:00 pm	Emily Witt	10:30–11:30 am	Michael DiPasquale
4:00–5:00 pm	Poster session	11:30–12:30 pm	Brandilyn Stigler
5:00–6:00 pm	Piotr Wojciechowski		
6:30 pm	Party		

POSTERS

- Rebekah Aduddell *The Natural Middle of a Complete Resolution*  
Tyler Anway *Resolutions via approximations of totally acyclic complexes*  
Laxman Bokati *How to Generate "Nice" Cubic Polynomials—with Rational Zeros and Rational Extrema*  
Alessandra Costantini *The Cohen-Macaulay property of the module of differentials*  
Benjamin Drabkin *Symbolic Defect and Cover Ideals*  
Nicholas Engel *Isomorphisms of Graded Skew Clifford Algebras*  
Fariba Khoshnasib *Bifurcations of Liouville Tori on different Lie Algebras*  
Vladik Kreinovich *Use of Symmetries in Economics: An Overview*  
Justin Lyle *The Auslander-Reiten Conjecture for Cohen-Macaulay Rings of Small Multiplicity*  
Bonifasio Menendez *The Quantum Space of the Heisenberg Lie Algebra*  
Cheng Meng *G-graded irreducibility and the index of reducibility*  
Nida Obatake *Hopf bifurcations in chemical reaction networks*  
Michael Perlman *Local cohomology with support in some orbit closures*  
Alexander Ruys de Perez *The Order Complex of a Neural Code*  
Alexandra Sobieska *Minimal Free Resolutions over Rational Normal Scrolls*  
Abu Thomas *Study of Resurgence and Waldschmidt constant of homogeneous ideals*  
Julio Urenda *When Revolutions Happen: Algebraic Explanation*  
Dwight Williams II *Infinite-dimensional Representations of  $\text{osp}(1|2n)$*

## Multigraded algebra over polynomial rings with real exponents

Ezra Miller

Commutative algebra over polynomial rings with real exponents has become important in the past decade because of applications in Topological Data Analysis. Specifically, persistent homology with multiple parameters results in nothing more or less than multigraded modules over real-exponent polynomial rings. This talk explains what that means (in detail, from scratch) and covers the fundamentals of (computational) commutative algebra in this setting: presentation, primary decomposition, syzygy theorem, and so on. One of the major concerns is what to do about the uncountably spectacular failure of noetherian hypotheses, especially given the computational motivations for thinking about real exponents in the first place. If you're interested, here are some warm-up exercises. In the two-variable real-exponent polynomial ring, find an ideal that is minimally generated by uncountably many monomials. Instruct a computer how to store your ideal. Compute a resolution of your ideal. Does your ideal have a primary decomposition? Show that the ideal of all real-exponent polynomials with constant term 0 is maximal and generated by countably many monomials, although it does not admit a minimal generating set. Do the same for the ideal of all real-exponent polynomials with no pure powers of (say) the first variable, but with "prime" instead of "maximal". It is not at all necessary to do these exercises to understand the talk, but they'll help you appreciate what's going on.



## Frobenius powers of ideals

Emily Witt

Given an ideal and a positive real parameter, there are many ways to construct a new ideal. For example, when the parameter is an integer, one may simply take the corresponding power of the ideal. Similarly, in prime characteristic, if the parameter is an integer power of the characteristic, then one may take the Frobenius power of the ideal. Further examples of this include the multiplier ideal construction from birational geometry, and the test ideal construction in prime characteristic. These constructions are known to be useful tools in measuring the singularities of the original ideal, and have recently been the subject of intense study. In this talk, we discuss a new construction in prime characteristic that "mimics" the usual Frobenius powers of an ideal. We will relate these "generalized Frobenius powers" to test ideals and multiplier ideals, and describe how they measure the singularities of generic polynomials.



## Full algebras of matrices and transitive systems

Piotr Wojciechowski

A subalgebra  $\mathcal{A}$  of  $M_n(\mathbb{R})$  is called *full* if for all  $i = 1, \dots, n$ ,  $E_{ii} \in \mathcal{A}$ . All such algebras are permutation-isomorphic to block lower-triangular matrices with corresponding subdiagonal blocks being either zero-blocks or full. Two full algebras are isomorphic if and only if they are permutation-isomorphic. A one-to-one correspondence is provided between the full algebras and transitive directed graphs. It is also proven that such algebras, if endowed with a lattice order, can be almost- $f$ - or  $d$ -algebras only if they are diagonal. Moreover, for every totally ordered field  $\mathbb{F}$ , if all  $E_{ii} > 0$  in a lattice-ordered full subalgebra  $\mathcal{A}$  of  $M_n(\mathbb{F})$ , then  $\mathcal{A}$  is isomorphic to the algebra ordered in the usual way. By a *transitive system* we understand a tuple  $(X, \mathfrak{R}, f, G)$  where  $X$  is a nonempty set,  $\mathfrak{R}$  is a reflexive and transitive relation on  $X$ ,  $G$  is a group and  $f$  is a *transitive function* defined on  $\mathfrak{R}$  into  $G$ . This concept emerged naturally in determining the isomorphisms of full algebras of matrices. Some examples and properties of the transitive systems will be presented, together with their connection with the graph theory.



## Szygies in and from geometric combinatorics

Anton Dochtermann

In this survey talk we discuss several instances where objects and constructions from geometric/topological combinatorics in fact encode underlying \*algebraic\* relations among monomial and binomial ideals. In one direction, polytopal complexes which parametrize combinatorial data (homomorphism complexes, generalized permutohedra, secondary polytopes) lead to minimal cellular resolutions. In another, combinatorial moves that preserve topological properties of simplicial complexes (elementary collapses) lead to a large class of ideals with linear resolutions, generalizing Froberg's theorem for edge ideals and leading to higher-dimensional notions of chordality. In many cases the underlying ideals are well-studied in the algebraic community (stable ideals, determinantal ideals, toppling ideals) and lead to new results regarding resolutions and Betti numbers. At the same time algebraic properties of these ideals lead to new insights regarding the underlying combinatorial objects.



## Asymptotic resurgence via integral closures and linear programs

Michael DiPasquale

The symbolic powers of an ideal  $I$ , denoted  $I^{(s)}$ , are an important geometric analogue of taking regular powers. There is significant interest in the *containment problem*; that is studying which pairs  $(r, s)$  satisfy that  $I^{(s)} \subset I^r$ . A celebrated result of Ein, Lazarsfeld, and Smith and Hochster and Huneke states that  $I^{(hr)} \subset I^r$  if  $I$  is an ideal with big height  $h$  in a regular ring. In an effort to quantify these containment results more precisely, the notions of resurgence and asymptotic resurgence of an ideal were introduced by Bocci and Harbourne and Guardo, Harbourne, and Van Tuyl. We show that the asymptotic resurgence of an ideal can be computed using integral closures, which leads to a characterization of asymptotic resurgence as the maximum of finitely many Waldschmidt-like constants. For monomial ideals these constants can be computed by solving linear programs over the symbolic polyhedron introduced by Cooper, Embree, Ha, and Hoefel. This makes it reasonable to compute the asymptotic resurgence of many monomial ideals, leading to some interesting examples related to combinatorial optimization where asymptotic resurgence and resurgence are different. This is joint work with Chris Francisco, Jeff Mermin, and Jay Schweig.



## Algebraic Model Selection and Identification using Groebner Bases

Brandilyn Stigler

Developing predictive models from large-scale experimental data is an important problem in biological data science. In certain settings, it may be advantageous to discretize the data to improve the signal-to-noise ratio or reduce data size, for example. While there are many classes of functions that can fit data from an underlying network, all data discretized over a finite field are fit by polynomials. A useful consequence is that polynomial models of such data can be written in terms of a standard monomial basis, by way of Groebner bases of zero-dimensional ideals, where each choice of standard monomials provides a different prediction regarding network structure. In this work, we use affine transformations to partition data sets into equivalence classes with the same sets of standard monomials. This partition reveals that data sets with unique bases are a so-called linear shift of a staircase. Implications of this work are guidelines for designing experiments which maximize information content and for determining data sets which yield unambiguous predictions.

