Department of Physics
Mechanics Qualifier Exam
9:00 am-12:00 pm
Aug. 18. 2025

Instructions:
1. Do not write your name. Write only your UTEP ID number.
2. Answer only 4 of the 6 questions given here. Each question is worth 25 points. Read each question very carefully before deciding which ones to answer. Please write below which four problems you want to be graded.

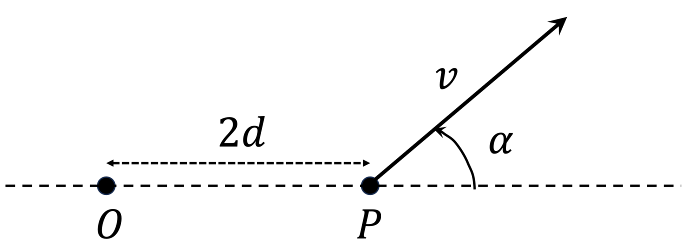
UTEP ID #:

Question numbers to be graded:

Prob.1 Two mass points m_1 and m_2 are connected by a string passing through a hole in a smooth table so that m_1 rests on the surface of the table (can(can move on surface of the table) the table) and m_2 hangs suspended. Consider the motion only so long as neither m_1 nor m_2 passes through the hole. Assuming m_2 moves only in a vertical line, what are the generalized coordinates for the system? Write the Lagrangian and the Lagrange's equations for the system.

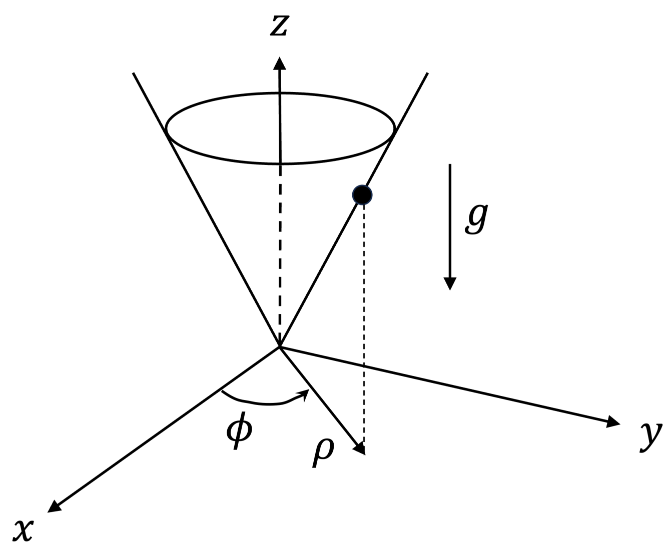
Prob.2 A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω.
(a) Obtain the Lagrange equations of motion assuming that the only external forces arise from gravity. What is the constant of motion?
(b) Show that if ω is greater than a critical value ω_0, there can be a stable stationary point on the hoop at a point other than the bottom, but that if $\omega < \omega_0$, the only stable stationary point for the particle is at the bottom of the hoop. What is the value of ω_0?

Prob.3 A mass particle m is moving under the attraction of an inverse square force of magnitude k/r^2. The particle was initially projected with speed $v = \sqrt{k/(2md)}$ from a point P a distance $2d$ from the force center O in a direction making an angle $\alpha = \pi/3$ with the line OP. See the figure below.



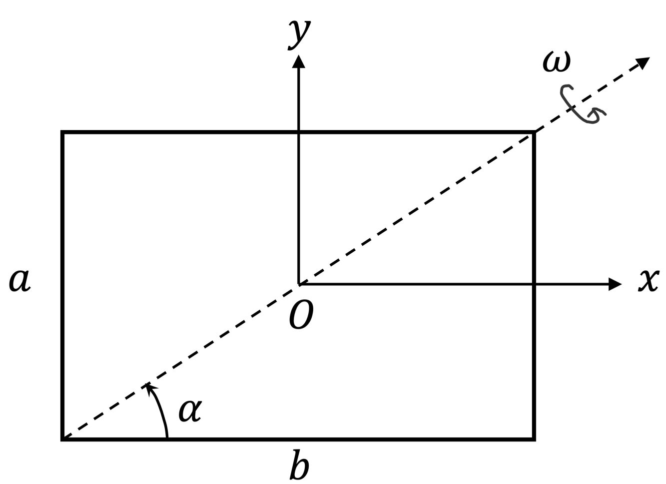
(a) Determine the energy of the particle, assuming that the potential energy of the particle at $r = \infty$ is zero.
(b) Determine the angular momentum of the particle.
(c) Determine the minimum and maximum distances of the particle from the force center in the subsequent motion. (*Hint: Using energy and angular momentum conservation, you can consider this as an effective one-dimensional motion.*)
(d) Determine the period of motion.

Prob.4 A particle is confined to, and moving on, a frictionless surface of revolution, a cone, defined by $z = a\rho$ in cylindrical coordinates (ρ, ϕ, z), for some positive constant a. The gravitational acceleration is g in the negative z direction. See the figure below.



(a) Find the Lagrangian in terms of ρ and ϕ.
(b) Find the Hamiltonian.
(c) Find the equations of motion.
(d) Determine whether it is possible to have a circular orbit about the z axis on this surface. If such an orbit exists, find its radius ρ_0 in terms of g, a, and the orbital angular frequency ω.

Prob. 5 Consider a uniform rectangular plate with total mass M, and side lengths a and b with $b > a$. Use the body frame with the origin of the coordinate system at the center of the plate, x-axis parallel to the longer side, and y-axis parallel to the shorter side. The plate rotates about one of its diagonals with a uniform angular velocity ω. See the figure below.



(a) Calculate the inertia tensor of the plate.
(b) What is the magnitude of the angular momentum of the plate? What is its direction?
(c) What is the torque acting on the plate?
(d) What is its kinetic energy?

Prob.6 For a one-dimensional motion with a generalized coordinate $q(>0)$ and the canonical momentum p, the transformation equations for a new set of coordinates are

$$Q = \ln(1+\sqrt{q}\cos p)$$
$$P = 2(1+\sqrt{q}\cos p)\sqrt{q}\sin p$$

where q and p are canonical variables.
(1) Using the Jacobian matrix and the symplectic condition, show that this transformation is a canonical transformation.
(2) Show that this transformation can be generated by the generating function $F = F_3(p,Q) + qp$ where

$$F_3(p,Q) = -\left[\exp(Q) - 1\right]^2 \tan p \ .$$

Useful equations for CM exam

Lagrange's equations of motion
For a system with generalized coordinates, $q_1, q_2, \cdots, q_n$, the Lagrange's equations are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0. \tag{1}$$

for $j = 1, \cdots, n$.

Useful integral

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \tag{2}$$

Inertia tensor
The elements of the inertia tensor for a continuous rigid body is defined as

$$I_{jk} = \int dv\, \rho(\mathbf{r}) \left(r^2 \delta_{jk} - r_j r_k\right) \tag{3}$$

where ρ is the mass density, and $j, k = x, y, z$.

Non-inertial frame of reference

$$\left(\frac{d\vec{A}}{dt}\right)_f = \left(\frac{d\vec{A}}{dt}\right)_r + \vec{\omega} \times \vec{A} \tag{4}$$

where $\vec{\omega}$ is the rotational velocity and the subscripts r and f refer to the non-inertial and inertial frames of reference, respectively.

Symplectic condition for a canonical transformation
The transformation between two sets of canonical coordinates $\{\eta_1, \cdots \eta_{2n}\}$ and $\{\zeta_1, \cdots \zeta_{2n}\}$ is a canonical transformation if and only if

$$\mathbf{M}^T \mathbf{J} \mathbf{M} = \mathbf{J} \tag{5}$$

where $\mathbf{M}$ is the Jacobian matrix ($2n \times 2n$) with matrix elements $M_{ij} = \partial \zeta_i / \partial \eta_j$ and $\mathbf{J}$ is the $2n \times 2n$ antisymmetric matrix,

$$\mathbf{J} = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{1} & \mathbf{0} \end{pmatrix} . \tag{6}$$

Canonical transformation by a generating function
The canonical transformation between two sets of canonical coordinates (q, p) and (Q, P) satisfies

$$P_i \dot{Q}_i - K + \frac{dF}{dt} = p_i \dot{q}_i - H \tag{7}$$

where F is the generating function.