

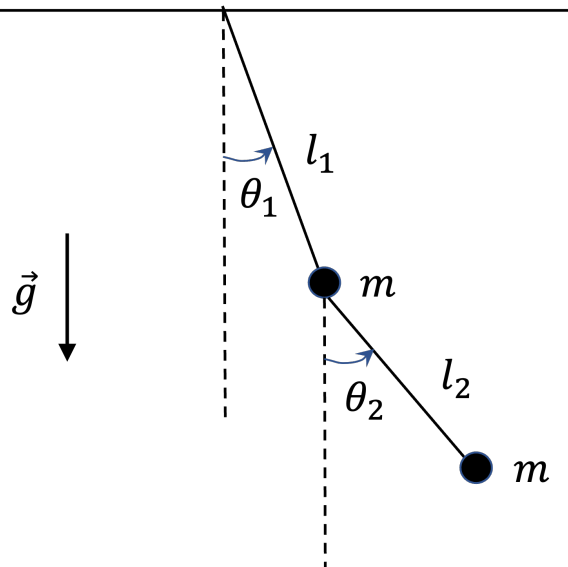
Department of Physics
Classical Mechanics Qualifier Exam
9:00 AM - 12:00 PM
Aug. 17 2026

INSTRUCTIONS:

1. Do not write your name anywhere. Write only your UTEP ID number.
2. Answer all the four questions given below.
3. Use separate pages for each question. Make sure to write your UTEP ID number on the right hand top corner of each page.
4. Show clearly all your work to receive proper credit.

UTEP ID #:

1. Consider a double pendulum that consists of two simple pendulums with different length. The upper pendulum with mass m and length l_1 is attached to a fixed support, and the lower pendulum with mass m and length l_2 is attached to the mass of the upper one. See the figure. The motion of this double pendulum is confined in a two-dimensional plane. The gravity acceleration is g .



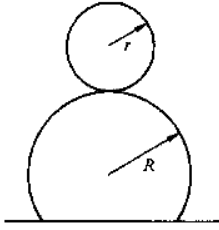
- (a) Calculate the kinetic energy T and the potential energy V in terms of the constants m , g , l_1 , l_2 , and the angles θ_1 , θ_2 .
- (b) Assuming the oscillation is small ($\theta_1, \theta_2 \ll 1$), write down the Lagrangian and the Lagrange's equations of motion.
- (c) Find all normal modes and corresponding mode frequencies ω (> 0) of the oscillation. You don't need to normalize the normal modes.

2. The classical interaction between two inert gas atoms, each of mass m , is given by the potential

$$V(r) = -\frac{2A}{r^6} + \frac{B}{r^{12}} \quad (1)$$

where $A, B > 0$ and $r = |\mathbf{r}_1 - \mathbf{r}_2|$. $\mathbf{r}_1$ and $\mathbf{r}_2$ are the positions of the atoms. Use the Cartesian center of mass coordinates, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2 = (X, Y, Z)$ and the relative position vector $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ in spherical coordinate system (r, θ, ϕ) as the generalized coordinates for this two-atom system.

- Give the Hamiltonian for the system of the two atoms.
 - Describe completely the classical state of this system with the lowest energy.
 - If the energy is slightly higher than the lowest in part (b), what are the possible frequencies of the motion of the system?
3. A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder.



4. A particle of mass m moves in an attractive central force field which is inversely proportional to the cube of the distance from the center.

$$\mathbf{F} = -\frac{k\hat{\mathbf{r}}}{r^3} \quad (2)$$

where k is a positive constant and $\hat{\mathbf{r}}$ is the unit vector in the radial direction. Using the polar coordinates for the plane of the motion,

- Find the Lagrangian and derive the equations of motion.
- Find the equation of motion for r in the equivalent one-dimensional problem with an effective potential. Plot the effective potential and describe the motion depending on the parameters.
- Find the orbit of this motion, i.e. $r(\theta)$. Compare the results with the qualitative description from (b).