Department of Physics
Mechanics Qualifier Exam
9:00 am-12:00 pm
Jan. 14 2026

Instructions:
Do not write your name anywhere. Write only your UTEP ID number.

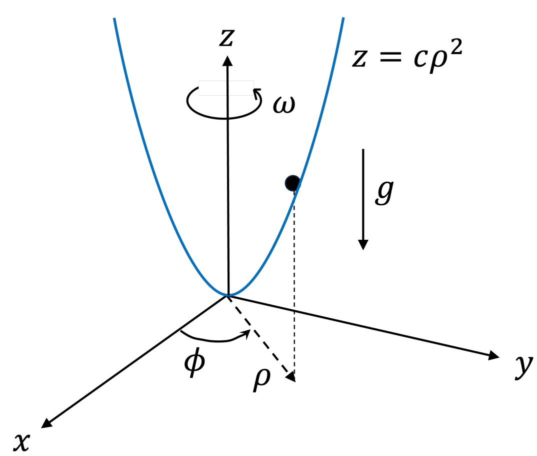
Answer all the four questions. Use separate pages for each question. Write your ID on the right hand top corner of each page.

Show clearly all your work to receive proper credit.

UTEP ID #:

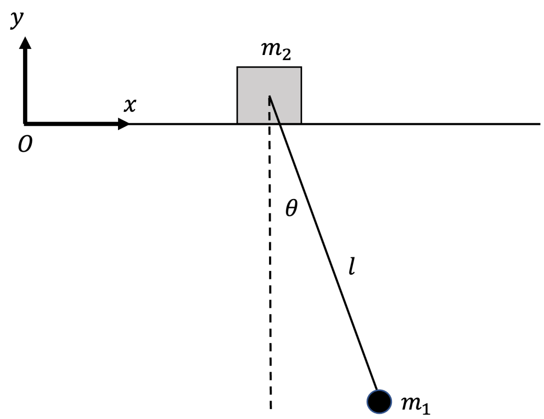
Prob.1 A particle of mass m moving in the potential field due to constant gravity $V(z) = mgz$ travels along the z-direction from the point $z = 0$ at $t = 0$ to $z = z_1$ at time t_1.
(a) Find the Lagrangian and write the action of the motion.
(b) Find the exact time dependence of the position of the particle by applying Hamilton's principle assuming it to be of the form $z(t) = At^2 + Bt + C$ and keeping the end points fixed.
(Do not solve Lagrange's equation of motion directly to find $z(t)$. You won't get any credit for it.)

Prob.2 A bead slides down a frictionless wire bent in the shape of a parabola $z = c\rho^2$ in the cylindrical coordinate system (ρ, ϕ, z). The wire is rotating around its vertical symmetry axis with a fixed angular velocity ω. The gravity is in the $-z$ direction with gravitational acceleration strength g. See the figure below.



(a) Write down the Lagrangian of the system.
(b) Write down Lagrange's equations of motion for the bead.
(c) Find the value of $\omega = \omega_{\rm cr}$ that allows the bead to rotate in a circle of fixed radius.
(d) Show that, if initially the bead is slightly perturbed from the bottom of the wire, it will oscillate around the bottom only if $\omega < \omega_{\rm cr}$, and find the oscillation frequency.

Prob.3. A block of mass m_2 is constrained to move without friction along a horizontal line. A simple pendulum of length ℓ and mass m_1 hangs from the block. See the figure below.



(a) Write the Lagrangian of this system.
(b) Find the normal modes and normal frequencies of the system assuming the pendulum angular displacement with respect to the vertical is small.

Prob.4 A particle of mass m moves in one dimension q in a potential energy field $V(q) = \frac{1}{2}m\omega^2 q^2$ and is retarded by a damping force $-2m\gamma\dot{q}$ proportional to its velocity.
(a) Write the equation of motion using the forces acting on the particle.
(b) Show that the equation of motion can also be obtained from the Lagrangian

$$L = \exp(2\gamma t)\left[\frac{1}{2}m\dot{q}^2 - V(q)\right] . \tag{1}$$

(c) Find the conjugate momentum and the Hamiltonian.
(d) Find the Hamiltonian $K(P,Q,t)$ for new canonical variables (P,Q) defined by

$$p = Pe^{\gamma t} , \quad Q = qe^{\gamma t} . \tag{2}$$

(*Hint:* This is a canonical transformation with the generating function $F = 0$. The new Hamiltonian $K(P,Q,t)$ must satisfy the canonical transformation relation, $p\dot{q} - H = P\dot{Q} - K$.)

Find the Hamilton's equations of motion for (P,Q), and find the frequency of motion for Q when $\omega > \gamma$.

Useful equations for CM exam

Hamilton's principle
The motion of a system from time t_1 to time t_2 is such that the time integral

$$I = \int_{t_1}^{t_2} L dt \tag{3}$$

where $L = T - V$ and with fixed end points, has a stationary value for the actual path of the system.

Lagrange's equations of motion
For a system with generalized coordinates, $q_1, q_2, \cdots, q_n$, the Lagrange's equations are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0. \tag{4}$$

for $j = 1, \cdots, n$.

Normal modes for small oscillations
For small oscillation problems, the Lagrangian of the system can be written as

$$L = \frac{1}{2} T_{ij} \dot{\eta}_i \dot{\eta}_j - \frac{1}{2} V_{ij} \eta_i \eta_j \tag{5}$$

where η_j's are the deviations of coordinates from the equilibrium positions. The normal frequency ω and the normal mode $\mathbf{a} = (\eta_1, ..., \eta_n)^T$ are determined by

$$\mathbf{Va} = \omega^2 \mathbf{a}^T \mathbf{Ta} \ , \tag{6}$$

where $\mathbf{V}$ and $\mathbf{T}$ are the matrix form of the potential and the kinetic energies with elements defined by the Lagrangian above. The standard normalization condition for the normal modes is

$$\mathbf{a}^T \mathbf{Ta} = 1 \ . \tag{7}$$

Hamilton's equations of motion
For a system with generalized coordinates, $q_1, q_2, \cdots, q_n$, and the conjugate momenta $p_1, p_2, \cdots, p_n$, the Hamilton's equations are

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \qquad \dot{p}_j = -\frac{\partial H}{\partial q_j} \ . \tag{8}$$