

Department of Physics
Electrodynamics Qualifier Exam
9:00 am-12:00 pm
August 20, 2026

INSTRUCTIONS:

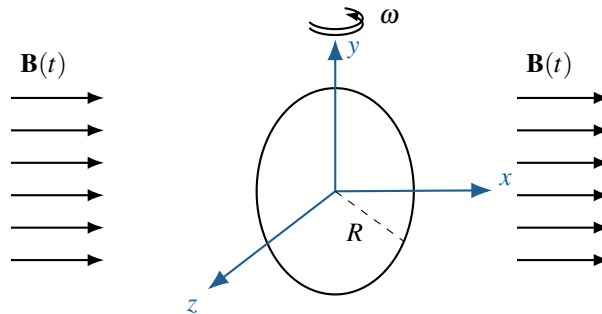
1. Do not write your name anywhere. Write only your UTEP ID number.
2. Answer all the four questions.
3. Use separate pages for each question. If you need additional sheets of paper, make sure to write your UTEP ID number on the right hand top corner of each page.
4. Show clearly all your work to receive proper credit.

UTEP ID #:

1. A circular loop of radius R is centered at the origin and is initially in the x - y plane, with its diameters along the x - and y -axes. The loop then rotates about the diameter lying along the y -axis with angular frequency ω . At the same time, a uniform magnetic field points along the x -direction and varies in time as

$$\vec{B}(t) = B_0 \cos(\omega t) \hat{x}.$$

Find the emf induced in the loop as a function of time.



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2. A thin spherical shell of radius a carries a fixed surface charge density given by

$$\sigma(\theta) = \sigma_0 \cos \theta,$$

where θ is the polar angle and σ_0 is a constant. There is vacuum both inside and outside the shell.

- (a) Determine the electrostatic potential $\Phi(r, \theta)$ everywhere and state clearly your boundary conditions.
- (b) Find the electric field in the interior region $r < a$ and provide its magnitude and direction.
- (c) Evaluate the E-field outside the shell and provide an expression for the dipole moment of the shell.

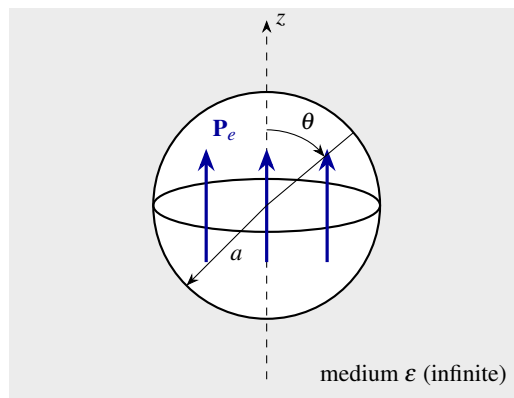
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3. Two long coaxial conducting cylinders, of inner radius a and outer radius b , are held vertically with their lower ends immersed in a large reservoir of liquid dielectric of permittivity ϵ and mass density ρ_m . A battery maintains a fixed potential difference V between the cylinders, and the liquid is observed to rise to a height h (above the reservoir level) in the annular gap. Neglect end effects, fringing fields, surface tension, and the susceptibility of air.

- (a) Show that the capacitance per unit length of a coaxial capacitor completely filled with a medium of permittivity ϵ is $C' = 2\pi\epsilon/\ln(b/a)$.
- (b) With the liquid risen to height h , write down the total capacitance $C(h)$ of a capacitor section of length L ($L > h$, measured from the reservoir surface).

4. A sphere of radius a is made of an electret material carrying a *permanent*, uniform (“frozen-in”) polarization $\mathbf{P}_e$ given by

$$\mathbf{P}_e = P_0 \hat{z},$$

Apart from this permanent polarization, the electret has no dielectric response, so inside the sphere $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}_e$. The sphere is embedded in an infinite linear, isotropic dielectric medium of permittivity ϵ . Assume there are no *free* charges anywhere.



- (a) Write down the boundary conditions at $r = a$.
- (b) Taking the potential to be of the form

$$\Phi(r, \theta) = \begin{cases} A \cos \theta & r \leq a \\ B \cos \theta / r^2 & r > a \end{cases}$$

and using your boundary conditions found in (a), determine the constants A and B .

- (c) Determine the electric field inside the sphere, and show that the field outside is that of a point dipole at the origin. Find the effective dipole moment.
- (d) Compute the bound surface charge density carried by the electret at $r = a$.
- (e) Show that in the limit $\epsilon \rightarrow \epsilon_0$ the electric field reduces to that of an isolated uniformly polarized sphere in vacuum, $\mathbf{E}_{\text{in}} = -\mathbf{P}_e/3\epsilon_0$.