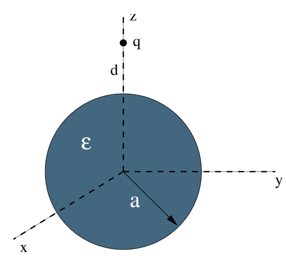
Department of Physics
Electrodynamics Qualifier Exam
9:00 am-12:00 pm
Aug. 20. 2025

Instructions:
1. Do not write your name. Write only your UTEP ID number.
2. Solve any 4 of the 6 problems in the test.
3. Read all problems carefully before deciding which ones to answer and write below the four problems you are submitting for grading.

UTEP ID #:

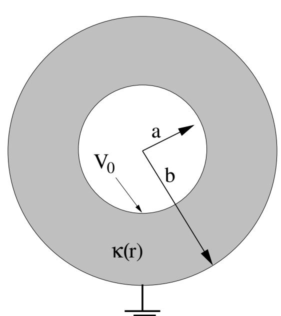
Question numbers to be graded:

1. A point charge q is placed a distance d from the center of an uncharged dielectric sphere of radius a and dielectric constant ϵ. In solving this problem consider the configuration shown in the figure below and assume the dielectric constant outside the sphere is ϵ_0.



(a) Calculate the electric potential inside and outside the sphere.
(b) Calculate the force on the point charge.

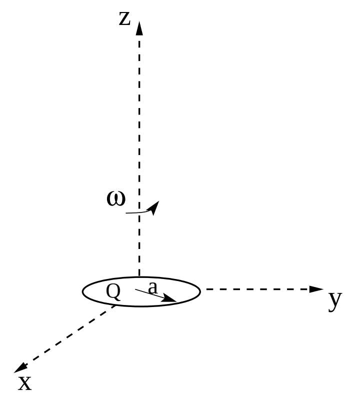
2. A spherical capacitor is constructed of two concentric metal spherical shells, of radii a and b and separated by a linear, isotropic but non-homogeneous dielectric material. In the dielectric, $D = \epsilon_0 \kappa \mathbf{E}$, where $\kappa = \kappa(r)$, such that $\mathbf{E}$ is constant. The inner metal sphere is kept at a potential V_0 while the outer metal sphere is grounded. The relative electric permittivity $\kappa = 3$ at $r = a$. There is no volume density of free charge in the dielectric. Express all results in terms of V_0, a, b, ϵ_0 and r.



(a) Write down the boundary conditions for the electric field $\mathbf{E}$ and displacement field $\mathbf{D}$ at $r = a$ and $r = b$.
(b) Find the relative permittivity $\kappa(r)$.
(c) Find the electric potential $\phi(r)$, the electric field $\mathbf{E}$, displacement field $\mathbf{D}$ and polarization vector $\mathbf{P}$ in the dielectric.

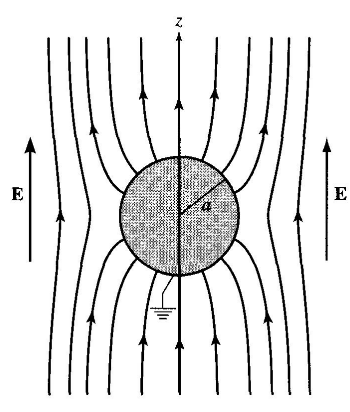
3. Consider a sphere of radius a with a uniform permanent magnetization $\mathbf{M} = M_0\hat{z}$.
(a) Calculate the magnetic induction $\mathbf{B}$ and the magnetic field $\mathbf{H}$ inside and outside the sphere.
(b) Find the magnetic dipole moment on the sphere.

4. A thin disk of radius a has a charge Q distributed uniformly on its surface. It rotates with constant angular frequency ω about an axis through its center and along its normal.

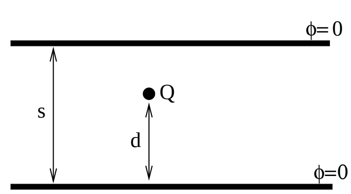


(a) Find the surface current $\mathbf{K}$ on the surface of the rotating disk.
(b) Find the magnetic field $\mathbf{B(0, 0, z)}$ on the z-axis a distance z from the center of the disk.
(c) Use your expression in (b) for $z >> a$, to find an expression for the magnetic dipole moment of the rotating disk.

5. A *grounded* conducting sphere of radius a is in an externally applied electric field pointing in the z-direction, $\mathbf{E} = E_0\hat{\mathbf{k}}$. Find an expression for the induced surface charge density $\sigma(\theta)$ on the surface of the sphere.



6. In the Millikan oil-drop experiment a small oil droplet carrying a few excess electrons is situated in the electric field between two charged parallel plates separated by a distance s. Consider the following analogous problem depicted in the figure below: a point charge Q is located between two grounded horizontal parallel plates



(a) Find the electric potential in the region in space between the plates.
(b) Calculate the total force on the charge Q due to the surface charge density between the plates.

Some Useful Formulae and Integrals

Biot-Savart Law: $d\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times (\vec{\mathbf{r}} - \vec{\mathbf{r}}')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|^3}$

Magnetic field due of point magnetic dipole $\mathbf{m}$:

$$\mathbf{B}(\mathbf{r}) \frac{\mu_0}{4\pi r^3} \left[3\hat{\mathbf{r}}(\mathbf{m} \cdot \hat{\mathbf{r}}) - \mathbf{m}\right]$$

SPHERICAL COORDINATES

$$\begin{aligned}
\hat{\mathbf{r}} &= \hat{\mathbf{i}} \sin\theta \cos\phi + \hat{\mathbf{j}} \sin\theta \sin\phi + \hat{\mathbf{k}} \cos\theta \\
\hat{\boldsymbol{\theta}} &= \hat{\mathbf{i}} \cos\theta \cos\phi + \cos\theta \sin\phi \hat{\mathbf{j}} - \sin\theta \hat{\mathbf{k}} \\
\hat{\boldsymbol{\phi}} &= -\hat{\mathbf{i}} \sin\phi + \hat{\mathbf{j}} \cos\phi
\end{aligned}$$

$$\boldsymbol{\nabla} f = \hat{\mathbf{r}} \frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi}$$

$$\boldsymbol{\nabla} \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$(\boldsymbol{\nabla} \times \mathbf{A})_r = \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial \theta}(\sin\theta\, A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right]$$

$$(\boldsymbol{\nabla} \times \mathbf{A})_\theta = \left[\frac{1}{r\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r}(r A_\phi)\right]$$

$$(\boldsymbol{\nabla} \times \mathbf{A})_\phi = \frac{1}{r} \left[\frac{\partial}{\partial r}(r\, A_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

INTEGRALS:

$$\int \frac{dx}{(x^2+a^2)} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x dx}{(x^2+a^2)} = \frac{1}{2} \ln\left[(x^2+a^2)\right]$$

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln\left[x + (x^2+a^2)^{1/2}\right]$$

$$\int \frac{x dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{(x^2+a^2)^{1/2}}$$

$$\int \frac{x dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$$

$$\int \frac{x dx}{(x+a)^{3/2}} = 2(x+a)^{1/2} + \frac{2a}{(x+a)^{1/2}}$$

Legendre Polynomials

$$\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= (3x^2-1)/2 \\
P_3(x) &= (5x^3-3x)/2
\end{aligned}$$

$$\int_{-1}^{1} P_l(x) P_{l'}(x) dx = \frac{2}{2l+1} \delta_{ll'}$$