Department of Physics
Quantum Mechanics Qualifier Exam
9:00 am-12:00 pm
Aug. 19. 2025

Instructions:
1. Do not write your name. Write only your UTEP ID number.
2. Solve any 4 of the 6 problems in the test.
3. Read all problems carefully before deciding which ones to answer and write below the four problems you are submitting for grading.

UTEP ID #:

Question numbers to be graded:

1. A free electron is at rest in a uniform magnetic field $\mathbf{B(t)} = \mathbf{B_0}\cos(\boldsymbol{\omega}\mathbf{t})\hat{\mathbf{k}}$.
(a) If time $t = 0$, the electron is in a state with spin up in the x-direction, find the probability that at a later time $t = T$, the electron is in a state with spin down in x-direction.
(b) What minimum value of the magnetic field amplitude B_0, will result in the probability of flipping the spin in the z-direction to be equal to 1, i.e., a complete flip in S_x?

2. An electron in a hydrogen atom has orbital angular momentum $l = 2$ and spin $s = 1/2$
Using the general raising and lowering operator formalism e.g.
$J_-|j,m_j\rangle = (J_x - iJ_y)|j,m_j\rangle = \hbar\sqrt{(j+m_j)(j-m_j+1)}|j,m_j-1\rangle$
Construct the linear combinations of $m_l m_s$ states which have
(a) $j = 5/2, m_j = 5/2 (j_z = m_j\hbar)$
(b) $j = 5/2, m_j = 3/2$
(c) $j = 3/2, m_j = 3/2$

3. Consider a complete set of (free particle) eigenfunctions of the Hamiltonian $H_0 = p^2/2m$, in a large *one-dimensional* box of length L with periodic boundary conditions:
$\psi_k(x) = \frac{1}{\sqrt{L}}e^{ikx}$, Eigenvalues: $E_k = \hbar^2k^2/2m$
A perturbation is added of the form
$V(x) = \lambda\cos(\mathrm{qx})$
where q is a mode of the box, i.e., $qL = 2n\pi, n =$ integer.
(a) Use perturbation theory to calculate the energy shifts due to $V(x)$ for all the states k in lowest non-vanishing order in λ. Use care in the region $k \approx \pm q/2$.
(b) Find the energy shift in the limit $k \to q/2$.

4. Consider a KCl crystal at room temperature that has some vacancies (missing atoms). An electron can occupy a vacancy defect (typically called $F-$ center) caused by an absent anion. An electron trapped at such a defect can absorb light in the visible spectrum, typically causing a normally transparent crystal to appear colored. If the electron moves freely in a cubic shaped defect whose volume is 1 Å^3, what is the longest wavelength of electromagnetic radiation absorbed by the electron.

5. Consider a particle of mass m that is moving in a one-dimensional oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$. If particle velocity v is much smaller than velocity of light c, then its kinetic energy is $\frac{p^2}{2m}$ and energy is $\frac{1}{2}\hbar\omega$. Suppose that the velocity increases significantly necessitating the corrections due to relativistic effects. Estimate the ground-level shift ΔE to the order $\frac{1}{c^2}$. (note: the relativistic kinetic energy is $T = E - mc^2$, where $E = \sqrt{m^2c^4 + p^2c^2}$).

6. Using a Gaussian function, $e^{-\beta r^2}$ as a trial wavefunction in a variational calculation, obtain an estimate of the energy of the hydrogen atom. Compare your answer with ground state solution

$$E_1 = -\frac{1}{2}\alpha^2 mc^2,$$

where $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$, is the fine-structure constant.

Some useful formulas

- $\int_0^\infty r^{2n} e^{-\frac{r^2}{a^2}} dr = \sqrt{\pi}\frac{(2n)!}{n!}(\frac{a}{2})^{2n+1}$
- $\int_0^\infty r^{2n+1} e^{-\frac{r^2}{a^2}} dr = \frac{n!}{2} a^{2n+2}$