

Department of Physics
Quantum Mechanics Qualifier Exam
9:00 am-12:00 pm
August 18, 2026

INSTRUCTIONS:

1. Do not write your name anywhere. Write only your UTEP ID number.
2. Answer all the four questions.
3. Use separate pages for each question. If you need additional sheets of paper, make sure to write your UTEP ID number on the right hand top corner of each page.
4. Show clearly all your work to receive proper credit.

UTEP ID #:

1. A quantum system has three-dimensional states spanned by the orthonormal basis $\{|1\rangle, |2\rangle, |3\rangle\}$. Two observables A and B can be represented in this basis by the matrices

$$\hat{A} = a \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad \hat{B} = b \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix},$$

where a and b are real constants. At $t = 0$, the system is in a state given by

$$|\psi\rangle = \frac{1}{\sqrt{6}}(|1\rangle + 2|2\rangle - i|3\rangle).$$

- (a) Verify that $\hat{A}$ is Hermitian, and find its eigenvalues and corresponding eigenvectors.
 - (b) If a measurement of observable A is made on $|\psi\rangle$, what possible values would one get and what would be their corresponding probabilities?
 - (c) Evaluate the expectation value of $\hat{A}$, $\langle A \rangle$ with respect to state $|\psi\rangle$ and also its uncertainty, $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$.
 - (d) A measurement of A is made and it yields the smallest eigenvalue of $\hat{A}$. Immediately afterwards, $\hat{B}$ is measured. Find the possible resulting values of this measurement and their corresponding probabilities.
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2. A one-dimensional harmonic oscillator of mass m and frequency ω is subject to a constant external force F , so that

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - F\hat{x}.$$

- (a) Write the Heisenberg equations of motion for $\hat{x}(t)$ and $\hat{p}(t)$ and solve them exactly in terms of the values of $\hat{x}$ and $\hat{p}$ at $t = 0$, namely $\hat{x}(0)$ and $\hat{p}(0)$.
 - (b) Evaluate the commutator $[\hat{x}(t), \hat{x}(0)]$ and show that it vanishes at times $t = n\pi/\omega$. Explain this result.
 - (c) At $t = 0$ the system is in the ground state of the *unperturbed* ($F = 0$) oscillator. Compute $\langle \hat{x}(t) \rangle$ and $\langle \hat{p}(t) \rangle$.
 - (d) Show that $\Delta x(t)$ is independent of time for this initial state. Explain what this says about the nature of the evolving state?
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3. A spin-1/2 particle with magnetic moment $\mu = \gamma \hat{\mathbf{S}}$ is in a uniform external magnetic field along the z -direction, $\mathbf{B} = B_0 \hat{z}$. The Hamiltonian can be written as

$$H = \omega \hat{S}_z,$$

with $\omega \equiv -\gamma B_0$. At $t = 0$ the particle is in the state $|S_x; +\rangle$.

- Find $|\psi(t)\rangle$ and compute $\langle \hat{S}_x \rangle$, $\langle \hat{S}_y \rangle$, and $\langle \hat{S}_z \rangle$ as functions of time.
 - Show directly from the Heisenberg equations that $d\langle \hat{\mathbf{S}} \rangle / dt = \gamma \langle \hat{\mathbf{S}} \rangle \times \mathbf{B}$, and comment on the correspondence with classical precession.
 - At the instant $T = \pi/2\omega$ the field is *suddenly* rotated to point along $\hat{x}$, with the same magnitude. Show that the state at $t = T$ is an eigenstate of $\hat{S}_y$, and find the probability of measuring $S_z = +\hbar/2$ at a later time $t = T + \tau$.
 - At what times τ is the probability found in (c) above, equal to unity?
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4. Consider a one-dimensional harmonic oscillator, whose Hamiltonian can be written in terms of ladder operators as

$$H_0 = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

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with $\hat{x} = \beta(\hat{a} + \hat{a}^\dagger)$ and $\beta = \sqrt{\hbar/2m\omega}$.

- A perturbation of the form $H_1 = \lambda \hat{x}^4$ is applied. Using the raising and lowering operators, find the first-order energy shift to the n -th energy level.
- Consider next, a perturbation of the form $H_1 = \lambda \hat{x}^3$. Compute the second-order shift to the ground state.
- The $\lambda \hat{x}^3$ potential is unbounded below for any $\lambda \neq 0$, so no true bound state exists. Explain what this implies about the perturbation series expansion.