# Department of Physics
# Quantum Mechanics Qualifier Exam
# 9:00 am-12:00 pm
# Jan. 15 2026

**Instructions:**
Do not write your name anywhere. Write only your UTEP ID number.

Answer all the four questions. Use separate pages for each question. Write your ID on the right hand top corner of each page.

Show clearly all your work to receive proper credit.

**UTEP ID #:**

1. An operator $\hat{A}$ representing an observable $A$ has two orthonormal eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with corresponding eigenvalues $a_1$ and $a_2$ respectively, i.e.

$$\hat{A}\,|\alpha_1\rangle = a_1\,|\alpha_1\rangle$$
$$\hat{A}\,|\alpha_2\rangle = a_2\,|\alpha_2\rangle$$

and $\langle\alpha_j\,|\alpha_i\rangle = \delta_{ij}$. Another operator $\hat{B}$ representing observable $B$ has two orthonormal eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with corresponding eigenvalues $b_1$ and $b_2$

$$\hat{B}\,|\beta_1\rangle = b_1\,|\beta_1\rangle$$
$$\hat{B}\,|\beta_2\rangle = b_2\,|\beta_2\rangle$$

Eigenstates $|\alpha_i\rangle$, $|\beta_i\rangle$ are related according to the relations

$$|\alpha_1\rangle = \frac{1}{5}\Big(3\,|\beta_1\rangle + 4\,|\beta_2\rangle\Big) \quad |\alpha_2\rangle = \frac{1}{5}\Big(4\,|\beta_1\rangle - 3\,|\beta_2\rangle\Big)$$

(a) Observable $A$ is measured, and the value $a_1$ is obtained. What is the state of the system immediately after this measurement?

(b) Following the $A$ measurement, observable $B$ is measured. What would be the possible results of this second measurement and what are their probabilities?

(c) Right after the $B$ measurement, observable $A$ is measured again. What would be the probability of getting the value $a_1$ in this measurement?

2. Consider a quantum system with just three linearly independent states, whose original Hamiltonian in matrix representation is given by

$$H_0 = E_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

A perturbation of the form

$$H_1 = \varepsilon \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

is added to the system, where $\varepsilon \ll E_0$.

(a) Solve for the exact eigenvalues of $H = H_0 + H_1$. Expand each eigenvalue as a power series up to second order in $\varepsilon$.

(b) Use non-degenerate perturbation theory to find the approximate eigenvalues for the state corresponding to the non-degenerate eigenvector of $H_0$.

(c) Using degenerate perturbation theory to find the first order correction to the initially degenerate states of $H_0$.

3. A particle is in the ground state of an infinite square well with walls at $x = 0$ and $x = a$. Suddenly the right wall moves from $x = a$ to $x = 2a$. If the energy of the particle is measured after the wall expansion, what will be the most probable value of the energy measurement and what would be the probability of getting this result?

4. Consider a one-dimensional quantum harmonic oscillator. Write down its Hamiltonian in both position space and momentum space. The ground-state wave function in position space is given by

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

Determine the corresponding ground-state wave function in momentum space.

## Some Useful Formulae and Integrals

$$\alpha(\text{Fine Structure Constant}) = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{\hbar c} = 1/137.036$$

$$a_0(\text{Bohr radius}) = \frac{\hbar}{mc\alpha} = 0.529\mathring{A}$$

$$\mu_B(\text{Bohr magneton}) = \frac{e\hbar}{2m_e} = 0.5788\times 10^{-4}\text{eV/T}$$

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Raising and lowering operators for the 1D harmonic oscillator:

$$\hat{A}^+\,|n\rangle = \sqrt{(n+1)}\;|n+1\rangle\,; \quad \hat{A}\,|n\rangle = \sqrt{(n)}\;|n-1\rangle$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{A}^+ + \hat{A}); \quad \hat{p} = \frac{i}{2}\sqrt{2m\hbar\omega}\,(\hat{A}^+ - \hat{A})$$

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Effective Hydrogen atom Hamiltonian for radial wave function:

$$H = -\frac{\hbar^2}{2m}\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right] + \frac{\hbar^2}{2mr^2}\ell(\ell+1) - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

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Perturbation theory: second order correction to the energy due to perturbation $V_1$:

$$E_n^{(2)} = \sum_{m\neq n}\frac{|\langle m|V_1|n\rangle|^2}{E_n^{(0)} - E_m^{(0)}}, \text{ where } |n\rangle \text{ are the eigenstates of the unperturbed hamiltonian}$$

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Hermite Polynomials

$$H_0(x) = 1; H_1(x) = 2x; H_2(x) = 4x^2 - 2; H_3(x) = 8x^3 - 12x$$

Normalized time-independent eigenfunctions of 1D harmonic oscillator:

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\zeta)e^{-\zeta^2/2}$$

where $\zeta = (m\omega/\hbar)^{1/2}x$

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INTEGRALS:

$$\int_{-\infty}^{\infty} e^{i(k-k')x}dx = 2\pi\delta(k-k')$$

$$\int_0^{\infty} x^n e^{-x}dx = n!$$

$$\int_0^{\infty} e^{-ax^2}dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^{\infty} xe^{-ax^2}dx = \frac{1}{2a}$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

One-dimensional Fourier transform:

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx} dk$$

Trigonometric relations:

$$cos(A \pm B) = cosAcosB \mp sinAsinB$$