

Department of Physics  
 Statistical Mechanics Qualifier Exam  
 9:00 am-12:00 pm  
 Aug. 21. 2025

**Instructions:**

1. Do not write your name. Write only your UTEP ID number.
2. Answer only 4 of the 6 questions given here. Each question is worth 25 points. Read each question very carefully before deciding which ones to answer. Please write below which four problems you want to be graded.

UTEP ID #:

**Question numbers to be graded:**

1. A system with two energy levels  $E_0$  and  $E_1$  system is populated by  $N$  particles at temperature  $T$  according to classical statistics.
  - (a) Obtain the average energy per particle as  $T \rightarrow 0$  and  $T \rightarrow \infty$ .
  - (b) Obtain the specific heat in the limits  $T \rightarrow 0$  and  $T \rightarrow \infty$ .
  
2. Consider a simple system of  $N$  almost noninteracting particles that can only occupy two energy levels  $\epsilon_- = -\epsilon_0$  and  $\epsilon_+ = \epsilon_0$ . Let  $N_{+/-}$  be the number of particles in the state  $\epsilon_{+/-}$ . Then the total number of particles is  $N = N_+ + N_-$  and the total energy of the system is  $E = N_+ \epsilon_+ + N_- \epsilon_- = (N_+ - N_-) \epsilon_0$ .
  - (a) Evaluate the entropy of the system  $S = k \ln \Omega$ . (You may assume that  $N$  is large and make use of the Stirling formula.)
  - (b) Evaluate the temperature of the system  $T = (\partial E / \partial S)_N$  and show that for  $T$  to be positive  $E$  must be negative.
  
3. A cylinder of volume  $V$  and height  $L$  contains a homogeneous monatomic ideal gas of  $N$  particles each of mass  $m$ . The cylinder is placed in a uniform external gravitational field which acts in the  $z$ -direction. The gas is in thermal equilibrium at a temperature  $T$ . The Hamiltonian of a single particle can be written as  $H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + mgz$ .
  - (a) Calculate the free energy and mean internal energy of this system.
  - (b) Find the particle density and the pressure as a function of  $z$ .
  
4. A one dimensional quantum harmonic oscillator is in thermal equilibrium with a heat bath at temperature  $T$ .
  - (a) What is the mean value  $\bar{E}$  energy of the oscillator as a function of  $T$ ?
  - (b) What are the root mean square fluctuations  $\Delta E$ , about  $\bar{E}$ ?
  - (c) In the limit  $k_B T \ll \hbar \omega$  and  $k_B T \gg \hbar \omega$ , what is the behavior of  $\Delta E$  and  $\bar{E}$ ?
  
5. Consider a classical ideal gas of  $N$  identical polar molecules. Each molecule has a permanent magnetic dipole moment  $\mu$  along its axis. The gas is at a temperature  $T$  in a uniform magnetic field  $\vec{B}$ . The Hamiltonian of a single molecule is given by

$$H = \frac{p_r^2}{2M} + \frac{p_\theta^2}{2I} + \frac{p_\phi^2}{2I \sin^2 \theta} - \mu B \cos \theta,$$

where  $M$ ,  $\vec{p}$  and  $I$  are the total mass, momentum and moment of inertia respectively. The magnetic field  $\vec{B}$  is chosen point along the  $z$ -direction and  $\theta$  is the angle between the magnetic moment  $\vec{\mu}$  and  $\vec{B}$ . Calculate the average magnetic moment (magnetization) of the gas and the paramagnetic susceptibility per molecule.

6. A rarefied gas is contained in a vessel of volume  $V$  at pressure  $p$ . Assume the velocity distribution of the particles in the gas is Maxwellian.
- (a) calculate the rate at which the gas flows out of the vessel into a vacuum, through a small hole in one of the walls, of area  $A$ .
  - (b) Taking the wall with the hole to be in the  $y$ - $z$  plane, find the velocity distribution in the  $x$ -direction of the gas molecules moving out of the hole.

**SOME USEFUL FORMULAE AND INTEGRALS**

Maxwell-Boltzmann velocity distribution:

$$f(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(\frac{-mv^2}{2k_B T}\right) \quad (1)$$

Fermi-Dirac distribution function

$$f_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1} \quad (2)$$

Bose-Einstein distribution function

$$f_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1} \quad (3)$$

$$dx dy dz = r^2 \sin\theta dr d\theta d\phi \quad (4)$$

$$\int_{-\infty}^{\infty} e^{i(k-k')x} dx = 2\pi\delta(k-k')$$

$$\int_0^{\infty} x^n e^{-x} dx = n!$$

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}$$

$$\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a} \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^{\infty} x^3 e^{-ax^2} dx = \frac{1}{2a^2}$$

$$\int_0^{\infty} x^4 e^{-ax^2} dx = \frac{3}{8a^2} \left(\frac{\pi}{a}\right)^{1/2}$$