

Department of Physics
Statistical Mechanics Qualifier Exam
9:00 am-12:00 pm
August 21, 2026

INSTRUCTIONS:

- 1. Do not write your name anywhere. Write only your UTEP ID number.**
- 2. Answer all the four questions.**
- 3. Use separate pages for each question. If you need additional sheets of paper, make sure to write your UTEP ID number on the right hand top corner of each page.**
- 4. Show clearly all your work to receive proper credit.**

UTEP ID #:

1. A sample of mineral oil is placed in an external magnetic field H . Each proton has spin $\frac{1}{2}$ and a magnetic moment μ ; it can, therefore, have two possible energies

$$\varepsilon = \pm\mu H, \quad (1)$$

corresponding to the two possible orientations of its spin. An applied radio-frequency field can induce transitions between these two energy levels if its frequency ν satisfies the Bohr condition

$$h\nu = 2\mu H. \quad (2)$$

The power absorbed from this radiation field is then proportional to the difference in the number of nuclei occupying these two energy levels.

Assume that the protons in the mineral oil are in thermal equilibrium at a temperature T that is sufficiently high such that

$$\mu H \ll kT. \quad (3)$$

How does the absorbed power depend on the temperature T of the sample?

2. Two systems are allowed to exchange particles but are kept at fixed temperature and volume. The total particle number is fixed:

$$N = N_1 + N_2.$$

The total Helmholtz free energy is

$$F = F_1(T, V_1, N_1) + F_2(T, V_2, N_2).$$

- (i) Show that equilibrium requires

$$\mu_1 = \mu_2.$$

- (ii) Explain why particle transfer stops when the chemical potentials become equal.

3. Consider a simple system of N almost noninteracting particles that can only occupy two energy levels $\epsilon_- = -\epsilon_0$ and $\epsilon_+ = \epsilon_0$. Let $N_{+/-}$ be the number of particles in the state $\epsilon_{+/-}$. Then the total number of particles is $N = N_+ + N_-$ and the total energy of the system is $E = N_+\epsilon_+ + N_-\epsilon_- = (N_+ - N_-)\epsilon_0$.
- (a) Find the *microcanonical* partition function $\Omega(N, E)$. [Hint. Count the number of microstates for a system with fixed numbers N_+ and N_- and express your result in terms of N , E , and ϵ_0 .]
- (b) Evaluate the entropy of the system $S = k \ln \Omega$. (You may assume that N is large and make use of the Stirling formula.)
- (c) Evaluate the temperature of the system $T = (\partial E / \partial S)_N$ and show that for T to be positive E must be negative.
4. Consider a classical ideal gas of N identical polar molecules. Each molecule has a permanent magnetic dipole moment μ along its axis. The gas is at temperature T in a uniform magnetic field $\vec{B}$. The Hamiltonian of a single molecule is:

$$H = \frac{p^2}{2M} + \frac{p_\theta^2}{2I} + \frac{p_\phi^2}{2I \sin^2 \theta} - \mu B \cos \theta,$$

where M , $\vec{p}$, and I are the mass, momentum, and moment of inertia, respectively. The magnetic field $\vec{B}$ points along the z -axis, and θ is the angle between $\vec{\mu}$ and $\vec{B}$.

- (a) Calculate the average magnetic moment (magnetization) of the gas.
- (b) Calculate the paramagnetic susceptibility per molecule.